1.13.2014 ECE 421 ①

To understand aliasing and periodicity in discrete time signals, consider these analog signals:

$x_{1a}(t) = \sin(0.01\pi t)$,

$x_{2a}(t) = \sin(2.01\pi t)$.

You can see that the first signal has period 200,

$$\begin{aligned} x_{1a}(t+200) &= \sin(0.01\pi(t+200)) \\ &= \sin(0.01\pi t + 2\pi) \\ &= \sin(0.01\pi t) \\ &= x_{1a}(t). \end{aligned}$$

period = 200

Similarly, the second signal has a shorter period:

$$\begin{aligned} x_{2a}\left(t + \tfrac{2}{2.01}\right) &= \sin\left(2.01\pi\left(t+\tfrac{2}{2.01}\right)\right) \\ &= \sin(2.01\pi t + 2\pi) \\ &= x_{2a}(t). \end{aligned}$$

period $= \frac{2}{2.01}$

Plotted next to each other, $x_{2a}(t)$ oscillates 201 times (!) faster than $x_{1a}(t)$.

But now let's sample them with sampling rate $F_s = 1$, meaning 1 time unit per sample. In other words, $t = n$ for sample n:

$$\begin{aligned} x_1(n) &= x_{1a}(t=n) \\ &= \sin(0.01\pi n). \end{aligned}$$

sampling

similarly,

$$\begin{aligned} x_2(n) &= x_{2a}(t=n) \\ &= \sin(2.01\pi n) \\ &= \sin(2\pi n + 0.01\pi n) \\ &= \sin(0.01\pi n) \\ &= x_1(n). \end{aligned}$$

adding a multiple of 2π has no effect

This is aliasing. The discrete time signals look the same.

Problem 1.2

This problem appears on Handout #2, page 10.

(a) $\cos(0.1\pi n) \overset{\text{add period } P}{=} \cos(0.01\pi(n+P))$

$= \cos(0.01\pi n + 0.01\pi p)$

we need

$$0.01\pi p = 2\pi \ell,$$

where ℓ is integer, i.e., $\ell \in \mathbb{Z}$.

This is solved using

$$P = 200,$$

and then $0.01\pi P = 2\pi$, which is one cycle.

The period is 200.

(b) $\cos\left(\pi \frac{30n}{105}\right) = \cos\left(\frac{\pi 30}{105}(n+P)\right)$

$\pi \frac{30}{105} p = 2\pi \ell$

$\cancel{\pi} \cdot \frac{\overset{3}{\cancel{6}} \cdot \cancel{5}}{21 \cdot \cancel{5}} \cdot p = \cancel{\pi} \cdot \cancel{2}\ell \implies p \cdot \frac{3}{21} = \ell$

$\implies p = 7\ell.$

How about a period of $P=7$, which leads to one cycle? That works, because

$$\frac{30}{105}\pi \cdot p = \frac{30}{105}\pi \cdot 7 = \frac{210}{105}\pi = 2\pi.$$