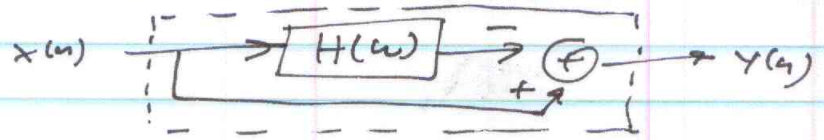
3.18.2014

supplement to handout #13

Problem 5.57

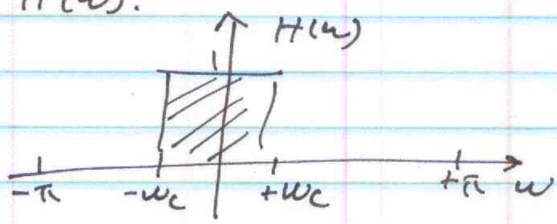
consider the following system



Determine its frequency response:

(a) $H(w)$ is lowpass with cutoff w_c.

Let's look at $H(w)$.

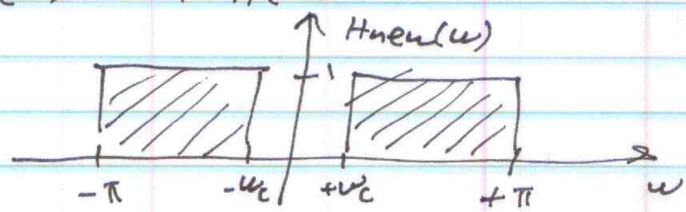


The new system is

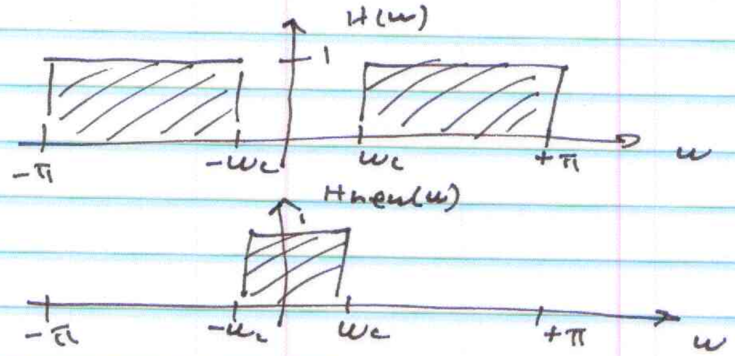
$$Y(w) = -X(w)\cdot H(w) + X(w)$$
$$= X(w)\,[1 - H(w)].$$

Therefore,

$$H_{new}(w) = 1 - H(w).$$



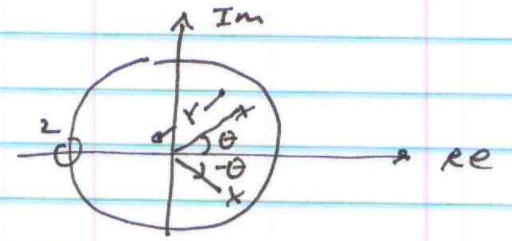
(b) $H(w)$ is highpass with cutoff w_c.



Now the new filter, $H_{new}(w)$, is lowpass.

New example

Let's construct a "reasonable" lowpass. It'll have poles at frequencies $\pm\theta$ (both are complex conjugates), and a double zero at -1 (on unit circle).



The radii of the poles will be r, and so

$$H(z) = \frac{(z+1)^2}{(z-re^{j\theta})(z-re^{-j\theta})} \cdot G.$$

Reasonable constraints are,

$$H(0) = H(\theta) = 1.$$

Let's start with constraint #1.

$w=0$ implies $z=e^{jw}=e^{0}=1.$

$$H(z=1)=\frac{(1+1)^2}{(1-re^{j\theta})(1-re^{-j\theta})}\cdot G=1$$

$$\begin{aligned}4G &= (1-re^{j\theta})(1-re^{-j\theta})\\ &= 1-r(e^{j\theta}+e^{-j\theta})+r^2\\ &= 1+r^2-2r\cos(\theta). \quad (*)\end{aligned}$$

constraint #2:

$w=\theta \rightarrow z=e^{j\theta}.$

$$H(z=e^{j\theta})=\frac{(e^{j\theta}+1)^2\cdot G}{(e^{j\theta}-re^{j\theta})(e^{j\theta}-re^{-j\theta})}=1.$$

$$(e^{j\theta}+1)^2\cdot G=(e^{j\theta}(1-r))\cdot(e^{j\theta}-re^{-j\theta})$$

$(**)$

we have two parameters, G and r.
These should (hopefully) be solvable from constraints $(*)$ and $(**)$ above.