3.30.2014

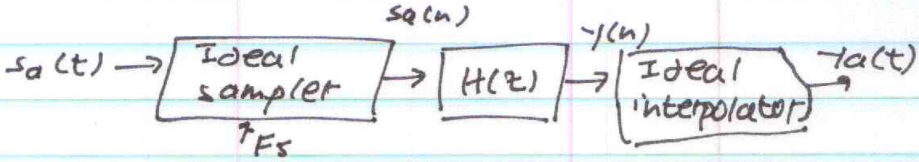
supplement to Handout #16

Problem 6.13

A continuous - time signal $x_a(t)$ with bandwidth B and its echo $x_a(t-\tau)$ arrive simultaneously at a TV receiver. The received analog signal

$$s_a(t) = x_a(t) + \alpha x_a(t-\tau), \quad |\alpha| < 1$$

is processed by the system below. Is it possible to specify Fs and $H(z)$ so that $y_a(t) = x_a(t)$?



To answer this question, we need to understand what ideal samplers and interpolators are.

Ideal sampler — multiply by impulse train (with frequency Fs), and then apply a lowpass.

Ideal interpolator — assign each y(n) sample on a delta (in continuous time), and then lowpass.

Let's realize that the Fourier transform of the echo is some complex exponent multiplied by $X_a(F)$ (multiplied by the attenuation α, of course).

Therefore, the bandwidth of $s_a(t)$ is identical to the bandwidth of $x_a(t)$. Because of this, $s_a(n)$ contains all the information in $s_a(t)$, meaning that $s_a(t)$ can be reconstructed from $s_a(n)$.

We can think of the relation between $x_a(t)$ and $s_a(t)$ as follows.

$$x_a(t) \rightarrow \boxed{G} \rightarrow s_a(t)$$

In the discrete time domain, we are sampling fast enough, and so:

$$x_a(n) \rightarrow \boxed{\tilde{G}} \rightarrow s_a(n)$$

All we need to do is identify $\tilde{G}$ (in discrete time) and apply its inverse,

$$\tilde{G}^{-1}(\omega) = \tilde{G}(\omega).$$