4. 12 · 2014

supplement to Handout # 19

Active learning example (pages 4-5)

$x(n) = (0 \quad 1 \quad 2 \quad 3)$.

we want to compute the DFT.

(1) $W_4 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = -j$

(2) $$W_4 = \begin{bmatrix} w_4^{0\cdot 0} & w_4^{0\cdot 1} & w_4^{0\cdot 2} & w_4^{0\cdot 3} \\ w_4^{1\cdot 0} & w_4^{1\cdot 1} & w_4^{1\cdot 2} & w_4^{1\cdot 3} \\ w_4^{2\cdot 0} & w_4^{2\cdot 1} & w_4^{2\cdot 2} & w_4^{2\cdot 3} \\ w_4^{3\cdot 0} & w_4^{3\cdot 1} & w_4^{3\cdot 2} & w_4^{3\cdot 3} \end{bmatrix}$$

$$= \begin{bmatrix} w_4^{0} & w_4^{0} & w_4^{0} & w_4^{0} \\ w_4^{0} & w_4^{1} & w_4^{2} & w_4^{3} \\ w_4^{0} & w_4^{2} & w_4^{4} & w_4^{6} \\ w_4^{0} & w_4^{3} & w_4^{6} & w_4^{9} \end{bmatrix}$$

(3) we now substitute $w_4 = -j$,

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix}$$

Note that $w_4^9 = w_4^4 \cdot w_4^4 \cdot w_4^1$, and

$w_N^N = (e^{-j\frac{2\pi}{N}})^N = e^{-j2\pi} = 1$.

Therefore $w_4^9 = w_4^1 = -j$.

(4) $$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot 0 + 1\cdot 1 + 1\cdot 2 + 1\cdot 3 \\ 1\cdot 0 - j\cdot 1 - 1\cdot 2 + j\cdot 3 \\ 1\cdot 0 - 1\cdot 1 + 1\cdot 2 - 1\cdot 3 \\ 1\cdot 0 + j\cdot 1 - 1\cdot 2 - j\cdot 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

This makes sense, because x is real-valued, and we have conjugate symmetries:

$$X(3) = X(1)^*$$
$$X(2) = X(2)^* \Rightarrow X(2) \in \mathbb{R}.$$

We can verify this in Matlab:

```
fft([0 1 2 3])
```