

# Homework 03 – Problem 3

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Let me describe Problem 3 in Homework 5 based on the parameters given to me by the software.

**Problem:** I was given a difference equation

$$y(n) - 0.25y(n-1) = 2x(n)$$

with an input

$$x(n) = 0.15^n u(n)$$

and initial conditions

$$y(-1) = 2.$$

We need to compute  $y(n)$ .

**Solution:** We compute the one sided  $z$  transform of the difference equation,

$$Y^+(z) - 0.25[z^{-1}Y^+(z) + y(-1)] = 2X^+(z). \quad (1)$$

We know that  $y(-1) = 2$ . Moreover, the one sided  $z$  transform of  $x(n)$  is

$$\begin{aligned} Z^+\{x(n)\} &= \sum_{n=0}^{+\infty} x(n)z^{-n} \\ &= \sum_{n=0}^{+\infty} 0.15^n u(n)z^{-n} \\ &= \sum_{n=0}^{+\infty} 0.15^n z^{-n} \\ &= \frac{1}{1 - 0.15z^{-1}}. \end{aligned}$$

Substituting into the one transform of the entire difference equation (1),

$$Y^+(z)[1 - 0.25z^{-1}] = \frac{2}{1 - 0.15z^{-1}} + 0.5,$$

where  $2X^+(z) = \frac{2}{1 - 0.15z^{-1}}$  and the 0.5 comes from the initial condition  $y(-1) = 2$  being multiplied by 0.25. Dividing by  $[1 - 0.25z^{-1}]$ ,

$$Y^+(z) = \left[ \frac{1}{1 - 0.25z^{-1}} \right] \left[ \frac{2}{1 - 0.15z^{-1}} + 0.5 \right]$$

$$\begin{aligned}
&= \frac{2}{(1 - 0.25z^{-1})(1 - 0.15z^{-1})} + \frac{0.5}{1 - 0.25z^{-1}} \\
&= \frac{2 + 0.5(1 - 0.15z^{-1})}{(1 - 0.25z^{-1})(1 - 0.15z^{-1})}.
\end{aligned}$$

A partial fraction expansion could be

$$Y^+(z) = \frac{a}{1 - 0.15z^{-1}} + \frac{b}{1 - 0.25z^{-1}}.$$

We see that

$$a(1 - 0.25z^{-1}) + b(1 - 0.15z^{-1}) = (a + b) + z^{-1}(-0.25a - 0.15b) = 2 + 0.5(1 - 0.15z^{-1}) = 2.5 - 0.075z^{-1}.$$

The constants  $a = -3$  and  $b = 5.5$  can be determined, for example using Matlab. Plugging them in,

$$Y^+(z) = \frac{-3}{1 - 0.15z^{-1}} + \frac{5.5}{1 - 0.25z^{-1}}.$$

Taking the inverse one sided transform yields

$$y(n) = -3(0.15)^n u(n) + 5.5(0.25)^n u(n).$$