Homework 07 – Problem 1

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Let me describe Problem 1 in Homework 7 based on the parameters given to me by the software.

Problem: We are given a periodic signal,

$$x(n) = 7\cos(2\pi\frac{13}{28}(n-12) - \frac{\pi}{4}).$$

1) We need to compute the period N. The period provides an integer number of 2π radians. Seeing 28 in the denominator, let's try that. If $N = 28$, then we have $2\pi 13$ radians, which is 13 cycles. Moreover, because 13 is a prime number, we cannot reduce the period N further by dividing it by some other integer.

2) We are asked to express $x(n)$ as a sum of exponentials. Recall that $\cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$, and so we see that the phase is $\theta = 2\pi\frac{13}{28}(n-12) - \frac{\pi}{4}$, and the sum of exponentials becomes

$$x(n) = \frac{7}{2}e^{j[2\pi\frac{13}{28}(n-12)-\frac{\pi}{4}]} + \frac{7}{2}e^{-j[2\pi\frac{13}{28}(n-12)-\frac{\pi}{4}]},$$

where $\frac{7}{2}$ is due to the 7 being in $x(n)$ being multipled by $\frac{1}{2}$ in each exponent.

3) Recall that the indices have the form

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}.$$

Therefore,

$$\begin{aligned} c_k &= \frac{1}{N}\sum_{n=0}^{N-1}\left[\frac{7}{2}e^{j[2\pi\frac{13}{28}(n-12)-\frac{\pi}{4}]} + \frac{7}{2}e^{-j[2\pi\frac{13}{28}(n-12)-\frac{\pi}{4}]}\right]e^{-j2\pi kn/N} \\ &= \frac{7}{2}\frac{1}{28}\sum_{n=0}^{27} e^{j2\pi\frac{13(n-12)-kn}{28}-j\frac{\pi}{4}} + e^{j2\pi\frac{-13(n-12)-kn}{28}+j\frac{\pi}{4}}. \end{aligned}$$

The first exponent is non-zero only when $k = 13$. The second exponent is non-zero only when $k = -13$. Note, however, that $k \in \{0, 1, \ldots, N-1\}$. The $k = -13$ corresponds to $k = 15$ after adding 28 (things are modulo N), and so $k_1 = 13$ and $k_2 = 15$.

4) The actual coefficients are computed as follows,

$$c_{13} = \frac{7}{2}e^{-j2\pi\frac{13\cdot 12}{28}-j\frac{\pi}{4}},$$

$$c_{15} = \frac{7}{2} e^{j2\pi \frac{15 \cdot 12}{28} + j\frac{\pi}{4}}.$$

Note that the $\frac{1}{28}$ term is canceled out by all 28 terms in each summation having the same value.