# Homework 08 – Problem 1

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Let me describe Problem 1 in Homework 8 based on the parameters given to me by the software.

**Problem:** We are given the following system,

$$y(n) = x(n) - x(n-1).$$

**1)** We want to compute $H(\omega)$. To do so, we take the $z$ transform of the difference equation,

$$Y(z) = X(z) - X(z)z^{-1},$$

which can be simplified as

$$Y(z) = X(z)[1 - z^{-1}].$$

We can now compute the transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-1}.$$

In the Fourier domain, we obtain $H(\omega)$ by substituting $z = e^{j\omega}$, and so

$$H(\omega) = 1 - e^{-j\omega}.$$

**2)** We want to compute $|H(\omega)|$. To do so, we take the square root of the sum of the squares of the real and imaginary parts of $H(\omega)$. Recall that $e^{-j\omega} = \cos(\omega) - j\sin(\omega)$, and so the real part is

$$Re\{H(\omega)\} = 1 - \cos(\omega),$$

and the imaginary part is

$$Im\{H(\omega)\} = \sin(\omega).$$

We can now compute the magnitude,

$$\begin{aligned} |H(\omega)| &= \sqrt{[Re\{H(\omega)\}]^2 + [Im\{H(\omega)\}]^2} \\ &= \sqrt{[1 - \cos(\omega)]^2 + [\sin(\omega)]^2} \\ &= \sqrt{1 - 2\cos(\omega) + \cos^2(\omega) + \sin^2(\omega)} \\ &= \sqrt{2 - 2\cos(\omega)}. \end{aligned}$$

**3)** The phase of $H(\omega)$ is computed by taking the arc tangent of the imaginary part divided by the real part,

$$< \{H(\omega)\} = \tan^{-1}\left(\frac{Im\{H(\omega)\}}{Re\{H(\omega)\}}\right) = \tan^{-1}\left(\frac{\sin(\omega)}{1-\cos(\omega)}\right).$$

**4)** We now have an input

$$x(n) = \cos(2\pi n) + \cos(\pi n).$$

To compute the ouput, we must compute $H(\omega)$ at the two frequencies present in $x(n)$, namely $\omega_1 = 2\pi$ and $\omega_2 = \pi$. Note that $\omega_1 = 2\pi$ is identical to $\omega_1 = 0$, and

$$H(\omega_1 = 0) = 1 - e^{-j\cdot 0} = 1 - 1 = 0.$$

Similarly,

$$H(\omega_2 = \pi) = 1 - e^{-j\cdot\pi} = 1 - (-1) = 2.$$

Multiplying each cosine in $x(n)$ by the amplitude that our LTI system magnifies it by,

$$y(n) = H(0)\cos(2\pi n) + H(\pi)\cos(\pi n) = 2\cos(\pi n).$$