Homework 2

Dror Baron

Problem 2 part a: An analog signal contains frequencies up to 14 KHz. What range of frequencies should we sample at?

Answer: We must sample above the Nyquist rate, which is double the highest frequency in the analog signal. In our case,

$$F_s = 2 \cdot 14KHz = 28KHz.$$

Problem 2 part b: Suppose we sample at $F_s = 7$ KHz. What frequency F_{1s} will a signal at $F_1 = 6$ KHz appear to be?

Answer: If F_1 is sampled above the Nyquist rate, meaning that $6KHz < \frac{1}{2}F_s = \frac{1}{2}7$ KHz, then there is no aliasing and $F_{1s} = F_1$ (this is not the case here). By contrast, if 6 KHz is sampled below the Nyquist rate (this is our situation), then there is aliasing. In this case, we must subtract $F_s = 7$ from F_1, possibly several times, until the resulting value is in the range $(-F_s/2, +F_s/2)$. If the resulting frequency within this range is positive, then we are done, else we take its absolute value.

In our case, there is aliasing, subtracting F_s from F_1 moves us to the range $(6-7)KHz = -1KHz \in (-F_s/2, +F_s/2)$, and we take its absolute value. Therefore, $F_{1s} = 1KHz$.

Problem 3 part a: We sample at $F_s = 19$ Hz, and the analog frequency is $F = 3$. What does the sampled cosine appear to be?

Answer: If the cosine at frequency $F = 3$ is sampled above its Nyquist rate, meaning that $F_s = 19Hz > 2F = 2 \cdot 3 = 6$ Hz, then there is no aliasing and the analog frequency is seen at $F = 3$. Else there is aliasing, we subtract F_s from F, possibly several times, until we have a frequency within the range $(-F_s/2, +F_s/2)$, and take its absolute value.

In our case, there is no aliasing the frequency is seen as $F = 3$ Hz.

Problem 3 part c: Here the analog frequency is $F = 12.5$ Hz.

Answer: In this case, there is aliasing, subtracting $F_s = 19$ Hz from $F = 12.5$ Hz moves us to the range $12.5 - 19Hz = -6.5Hz \in (-F_s/2, +F_s/2)$, and we take its absolute value. Therefore, the frequency is seen as 6.5 Hz.

Problem 4: The input signal is $3 \cdot \cos(30 \cdot 2\pi t + \pi/2) + 5 \cdot \cos(200 \cdot 2\pi t + \pi/4)$. We run an A/D converter at 300 Hz, and then a D/A at 1800 Hz. We want to express the analog output.

Answer: There are two steps. In the first A/D step, the time domain t is converted to discrete time samples n using $t = nT$. Noting that $T = 1/F_s = 1/300$ sec, the discrete time signal is

$$x(n) = 3 \cdot \cos(30 \cdot 2\pi n/300 + \pi/2) + 5 \cdot \cos(200 \cdot 2\pi n/300 + \pi/4).$$

We now have discrete time frequencies $f_1 = 30/300 = 1/10$ and $f_2 = 200/300 = 2/3$. We see that the first component is sampled without aliasing, $f_1 = 1/10 < \frac{1}{2}$. However, the second component suffers from aliasing, because $f_2 = 2/3 > \frac{1}{2}$. Instead, we replace f_2 by an aliased frequency $f_3 = 1/3 < \frac{1}{2}$, which is obtained by subtracting an integer from $f_2 = 2/3$ until it lies within $(-\frac{1}{2}, +\frac{1}{2})$, and then taking the absolute value, resulting in f_3. (Note that the sign of the phase shift, $\pi/4$, is also flipped, because we had to take the absolute value, which flipped the sign of the frequency.) In summary, the discrete time signal can be written as

$$x(n) = 3 \cdot \cos(1/10 \cdot 2\pi n + \pi/2) + 5 \cdot \cos(1/3 \cdot 2\pi n - \pi/4).$$

The last step entails running $x(n)$ through a digital to analog (D/A) converter. While the A/D step involved $t = nT = n/300$, the D/A step involves substituting $n = t1800$. Therefore, the output is

$$3 \cdot \cos(1/10 \cdot 1800 \cdot 2\pi t + \pi/2) + 5 \cdot \cos(1/3 \cdot 1800 \cdot 2\pi t - \pi/4).$$