Homework 4

Dror Baron

Problem 1: The signal takes on the following values,

$$x(n) = (-2, -3, -6, -2, 5, -9, 7, -6, -4, 8, 6),$$

where $n \in \{-5, \ldots, 5\}$. The z-transform is computed as follows,

$$\begin{aligned}
X(z) &= \sum_{n=-\infty}^{+\infty} x(n)z^{-n} \\
&= \sum_{n=-5}^{5} x(n)z^{-n} \\
&= x(-5)z^{-(-5)} + x(-4)z^{-(-4)} + x(-3)z^{-(-3)} + x(-2)z^{-(-2)} + x(-1)z^{-(-1)} + x(0)z^{-(0)} \\
&\quad + x(1)z^{-(1)} + x(2)z^{-(2)} + x(3)z^{-(3)} + x(4)z^{-(4)} + x(5)z^{-(5)} \\
&= -2z^5 - 3z^4 - 6z^3 - 2z^2 + 5z - 9 + 7z^{-1} - 6z^{-2} - 4z^{-3} + 8z^{-4} + 6z^{-5}.
\end{aligned}$$

Regarding the region of convergence (ROC), the key point to remember for finite duration signals is that there are three cases: (*i*) if the signal is causal, then ROC=$\{z : z \neq 0\}$; (*ii*) if the signal is anti-causal, then ROC=$\{z : z \neq \infty\}$; and (*iii*) if the signal is double-sided, then ROC=$\{z : z \neq 0, \infty\}$. In our case, the values of n starting from -5 and ending at 5 indicate that the signal is double-sided and ROC=$\{z : z \neq 0, \infty\}$.

Problem 2: The signal $x(n)$ can be partitioned into two components as follows,

$$\begin{aligned}
x(n) &= x_1(n) + x_2(n), \\
x_1(n) &= \left(\frac{1}{2}\right)^n u(n), \\
x_2(n) &= \left(\frac{2}{3}\right)^n u(-n-1).
\end{aligned}$$

The z-transforms of the two components are

$$\begin{aligned}
X_1(z) &= \frac{1}{1 - \frac{1}{2}z^{-1}}, \\
X_2(z) &= -\frac{1}{1 - \frac{2}{3}z^{-1}},
\end{aligned}$$

and the ROCs are $ROC_1 = \{z : |z| > \frac{1}{2}\}$ and $ROC_2 = \{z : |z| < \frac{2}{3}\}$. The z-transform of the entire signal follows using linearity,

$$X(z) = X_1(z) + X_2(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - \frac{2}{3}z^{-1}},$$

and the ROC is the intersection of ROC_1 and ROC_2. Because $\frac{1}{2} < \frac{2}{3}$, $ROC = \{z : \frac{1}{2} < |z| < \frac{2}{3}\}$.