

ECE 421 Introduction to Signal Processing

Final Exam – Spring 2016

April 29, 2016

Please remember to justify your answers carefully.

There are 10 pages in total on this test.

Name: _____ Student ID: _____

Question 1

(a) Compute the four-point discrete Fourier transform (DFT) $X_1(k)$ of the sequence $x_1(n) = [\underline{1}, 4, 2, 3]$ (the underline corresponds to time zero).

(b) We are now given that $x_2(n) = [\underline{1}, 1, 0, 0, 0, 1]$, $n = 0, \dots, 5$, and $X_3(k) = X_2(k)e^{-\frac{4j\pi k}{3}}$. Compute $x_3(n)$.

Question 2

Consider an analog resonator filter with system function,

$$H_a(s) = \frac{s + 2}{s^2 + 4s + 8}. \quad (1)$$

(a) Convert the analog filter into a digital infinite impulse response (IIR) filter using a bilinear transformation, where the digital filter is supposed to have a resonant frequency of $\omega_r = \pi/3$.

(b) To simplify calculations, in the rest of the question suppose that we obtained the following IIR system,

$$H(z) = \frac{1 - z^{-1}}{(1 + 0.5z^{-1})(1 - 2z^{-1})}. \quad (2)$$

- (i) Determine the value of $H(z)$ at frequencies $\omega_1 = 0$ and $\omega_2 = \pi$.
- (ii) Given that the system is causal, determine the region of convergence (ROC).
- (iii) Suppose that the system need no longer be casual. Can the system be stable? If yes, what is the corresponding ROC? If not, why?

Question 3

A digital notch filter is required to remove an undesirable 50Hz hum in some application. The sampling frequency used is $F_s = 180Hz$.

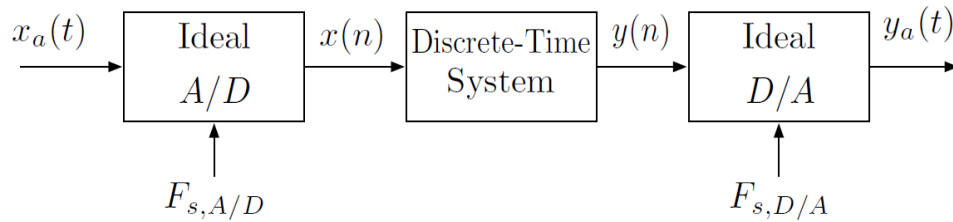
(a) Calculate the digital frequency ω_{int} (in radians per sample) of the hum.

(b) Design a second order notch filter $H(z)$ by placing two zeros symmetrically on the unit circle such that one zero is at the positive digital frequency of the interference signal, and the second zero is at the negative frequency. To make the notches narrow, we place two poles close to the zeros. The first pole has magnitude 0.9 at the positive frequency, and the second pole has magnitude 0.9 at the negative frequency. What is the system function $H(z)$?

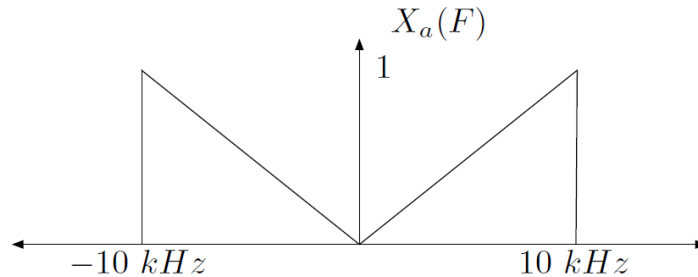
(c) Find the gain G of the filter such that $|GH(z)| = 1$ at zero frequency, i.e., $z = e^{j0}$.

Question 4

The following system is used to process an analog signal with a discrete-time system.



Suppose that $x_a(t)$ is bandlimited with $X_a(F) = 0$ for $|F| > 10$ kHz as shown in the figure below and suppose that the discrete-time system is an ideal low-pass filter with a cutoff frequency $\omega = \pi/4$ and gain 1.



(a) The analog signal $x_a(t)$ is sampled by an ideal analog to digital (A/D) converter with sampling rate $F_s = 10$ kHz. Draw $X(f)$, the Fourier transform of $x(n)$, the sampled signal in discrete time. Be sure to clearly label the horizontal axis, making note of the range of spectral content and aliased frequencies, if any.

(b) Draw $Y(f)$, the Fourier transform of $y(n)$, the discrete time output. As before, be sure to clearly label the horizontal axis.

(c) The discrete time output $y(n)$ is now passed through an ideal digital to analog (D/A) converter whose sampling rate is also $F_s = 10$ kHz. Draw $Y_a(F)$, the Fourier transform of $y_a(t)$, the analog output. As before, be sure to clearly label the horizontal axis.

Question 5

Consider a real valued input signal, $x_1(n) = [2, 4, 6, 8, 10, 12, 14]$, $n = 0, \dots, 6$. Four values of the DFT $X_1(k)$ are provided,

$$\begin{aligned} X_1(0) &= 56, \\ X_1(4) &= -7 - 1.6j, \\ X_1(5) &= -7 - 5.6j, \\ X_1(6) &= -7 - 14.5j. \end{aligned}$$

(a) Determine the missing values of $X_1(k)$ for $k \in \{1, 2, 3\}$. Remember to justify your answer.

(b) Determine the DFT $Y(k)$ of a signal $y(n)$ that results from filtering the signal $x_1(n)$ with an ideal highpass filter that keeps spectral content for $|\omega| \geq \frac{4\pi}{7}$.

(c) Consider another signal, $s(n) = [0, 0, 1, 0, 0, 0, 0]$, $n = 0, \dots, 6$ and its corresponding DFT, $S(k)$. Determine a sequence $x_2(n)$ such that $X_2(k) = X_1(k)S(k)$.

(d) Consider $x_3(n) = e^{-j\frac{6\pi n}{7}} x_1(n)$. Compute $X_3(k)$, the DFT of $x_3(n)$.

(e) We now want to use MATLAB functions *fft* and *ifft* to perform *linear* convolution of $x_1(n)$ and $s(n)$. Write MATLAB code for doing so without using the *conv* function. (Hint: you may want to use zero padding.)