ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2013 – Solution

February 7, 2014

<u>Question 1</u>

Part a) In order for the signal to be periodic, we would need both $\cos(\frac{4}{5}n)$ and $\sin(\frac{5}{3}n + \frac{1}{3})$ to be periodic. The cosine is not periodic, because $\frac{4}{5}$ is not of the form $2\pi q$ where q is a rational number. For the sine, first we can ignore the phase shift ot $+\frac{1}{3}$, because it has no impact on periodicity. Having ignored the phase, we would need $\frac{5}{3}$ to have the form $2\pi q'$ for some other rational number q'. We can see that the sine is not periodic either.

Part b) The only frequency is $F = 500$, which yields the form $\sin(2\pi F t) = \sin(1000\pi t)$. The Nyquist rate is $2F$, which is 1000 samples per unit time. An easy way to verify this result is that when we advance $\frac{1}{2F} = 0.001$ time units, the phase advances by $1000\pi \times 0.001 = \pi$ radians, which is half a cycle. If $x(t)$ is sampled at 800 Hz, the result is

$$x(n) = x\left(t = \frac{n}{800}\right) = 0.7 \sin\left(1000\pi \frac{n}{800}\right) = 0.7 \sin\left(\frac{5}{4}\pi n\right).$$

Note that $\sin\left(\frac{5}{4}\pi n\right) = \sin\left(-\frac{3}{4}\pi n\right)$, and so $x(n) = 0.7 \sin\left(-\frac{3}{4}\pi n\right)$.

Part c) It's easy to see the result with a table.

n	-4	-3	-2	-1	0	1
$-(n+2)$	2	1	0	-1	-2	-3
$x(-(n+2))$	0	-2	1	0	3	0
$2x(-(n+2))$	0	-4	2	0	6	0

In the sketch, I expected to see:

- Horizontal axis with an arrow pointing right.
- Vertical axis with an arrow pointing up.
- The horizontal axis is labeled with n (toward the right side), and near the top of the vertical axis $2x(-(n+2))$.
- To show the scale, need some numerical values along each axis.

Question 2

Part a) Not including a sketch – quite time consuming on a computer. I can make a handwritten sketch and scan it in, if this would help.
The idea is to break the difference equation into two parts. The input $x(n)$ enters on the left side, and there's a block on the left side that computes $v(n) = 3x(n) + 2x(n-1)$. The output of that block enters a second block that turns $v(n)$ into y as follows: $y(n) = 2y(n-1) + v(n)$. The output $y(n)$ will leave from the right side.
Part b) One way to solve this question is to first partition the block diagram into a right block and left block. Second, swap the order of the blocks (order of convolution doesn't matter). At this time, the blocks will have a form I style, which can be converted into a difference equation along the lines of Part a.

Question 3

Part a) Let us write the z transform of the difference equation,

$$Y(z) + 5z^{-1}Y(z) + 6z^{-1}Y(z) = X(z).$$

Next, can write

$$Y(z)[1 + 5z^{-1} + 6z^{-2}] = X(z).$$

the transfer function divides the output $Y(Z)$ by the input $X(z)$,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + 5z^{-1} + 6z^{-2}}.$$

Part b) The key task is to identify the zeros and poles.

$$\begin{aligned} H(z) &= \frac{1}{1 + 5z^{-1} + 6z^{-2}} \\ &= \frac{1}{(1 + 2z^{-1})(1 + 3z^{-1})} \\ &= \frac{z^2}{(z+2)(z+3)}. \end{aligned}$$

There are poles are at $p_1 = -2$ and $p_2 = -3$. Both poles are outside the unit circle. Note that there is a *double* zero. The system is stable, meaning that the unit circle must be inside the ROC. But with poles outside the unit circle, the responses must be anti-causal. The ROC for $p_1 = -2$ is $\mathrm{ROC}_1 = \{z : |z| < 2\}$, the ROC for $p_2 = -3$ is $\mathrm{ROC}_2 = \{z : |z| < 3\}$, and the entire ROC is the intersection, $\mathrm{ROC}_1 \cap \mathrm{ROC}_2 = \{z : |z| < 2\}$. (Note that "$\cap$" means intersection of sets.)
The sketch of the zeros and poles should contain:

- Horizontal axis with arrow at its right labeled with “Re(z).”
- Vertical axis with arrow at its top labeled with “Im(z).”
- Poles at $p_1 = -2$ and $p_2 = -3$. These are labeled by -2 and -3, respectively.
- A *double* zero at the origin, $z = 0$. The double nature of the zero is highlighted by the number 2 near the zero.
- The ROC is given by shading the circle of radius 2 centered around the origin. This circle should intersect the pole $p_1 = -2$.

As explained above, the system is anti-causal, because both poles are outside the unit circle yet it is stable.

Question 4

Part a) We have only covered the period so far. The Fourier transform will be studied in class soon.
To compute the period N, we must have $\frac{\pi N}{4} = 2\pi k$ for some integer k. It can be seen that $N = 8$, because each sample advances the phase by $\pi/4$ radians, and a cycle is 2π radians; 8 samples complete a cycle.