ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2014

February 12, 2014

Name: _____ Student ID: _____

Question 1

Consider the following signal,

$$x_a(t) = 0.4\sin(700\pi t) + 0.7\cos(1000\pi t).$$

1. What is the Nyquist rate required to sample $x_a(t)$?

2. Suppose that $x_a(t)$ from part 1 of this question is sampled at $F_s = 800$ samples per unit time. What is the resulting $x(n)$? In particular, provide detail about aliasing, including which old frequency gets mapped to which new frequency.

Question 2

Consider the following system,

$$y(n) = 3x(2n - 1).$$

1. Is the system $y(n)$ time invariant? Justify your answer.

2. Suppose that the input is $x(n) = \{3, 6, -2, \underset{\uparrow}{7}, 0, 3\}$.

   Using the system from part 1 of this question, derive the output $y(n)$.

Question 3

Consider the following difference equation,

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n-1).$$

1. Compute the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

2. Determine the zeros and poles of the system $H(z)$.

3. Suppose that the system is causal.

   (a) Determine the region of convergence (ROC).

   (b) Is the system stable?

   (c) Draw the poles, zeros, and ROC.

4. Suppose that the input signal is

$$x(n) = \left(\frac{1}{4}\right)^n u(n),$$

where $u(n)$ denotes the unit-step function. Using the transfer function $H(z)$ from part 1 of this question, express $X(z)$ and $Y(z)$, the z-transforms of the input and output signals, respectively.

Question 4

Consider an input signal $x(n)$ with the following z-transform,

$$X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})}.$$

1. Express $X(z)$ using a partial fraction expansion by writing $X(z)$ as a sum of two first-order systems,

   $$X(z) = \frac{a}{1 - \frac{1}{2}z^{-1}} + \frac{b}{1 + \frac{1}{2}z^{-1}}.$$

   In particular, compute the constants a and b.

2. Suppose that in part 1 of this question you got

   $$X(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{7}{1 + \frac{1}{2}z^{-1}},$$

   and suppose further that $x(n)$ is a causal signal ($x(n) = 0$ when $n < 0$). Compute $x(n)$.