ECE 421 Introduction to Signal Processing

Midterm 2 – Spring 2013

March 27, 2013

Name: _____ Student ID: _____

Question 1

Consider the signal

$$x(n) = \{1, 3, \underset{\uparrow}{-5}, 0, -1\}$$

with Fourier transform $X(\omega)$. Compute the following quantities: (Hint: you need not explicitly compute $X(\omega)$.)

(a) $X(0)$

(b) $\int_{-\pi}^{\pi} X(\omega) d\omega$

(c) $X(\pi)$

(d) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

Question 2

(a) Compute and sketch the magnitude response of the following system:

$$y(n) = \frac{1}{3}(x(n-1) + x(n-2) + x(n-3)).$$

(b) Consider the following inputs

$$(1) \qquad x(n) = \cos(\frac{\pi}{2}n),$$

$$(2) \qquad x(n) = 3\sin(\frac{3\pi}{4}n)u(n),$$

where $u(n)$ is the step function. Will either input have an output that is always 0? Why? (Hint: no need to calculate the output.)

Question 3

Given a continuous-time signal $x_a(t)$ with $X_a(F) = 0$ for $|F| > B$, determine the minimum sampling rate F_s for the following signals. Please explain your answer.

(a) $x_a^4(t)$

(b) $x_a(-t)x_a(t)$

Question 4

In this question, you will design a high pass filter with one zero and one pole. The zero will be at $\omega = 0$, and it will lie on the unit circle. The pole will be at $\omega = \pi$, and it will be inside the unit circle (we want the filter to be BIBO stable). This filter should have unit gain at $\omega = \pi$, i.e., $H(\pi) = 1$), and have a magnitude response of $\sqrt{0.1}$ at $\omega = \frac{\pi}{2}$, i.e., $H(\frac{\pi}{2}) = \sqrt{0.1}$. Please derive the system function and justify your design carefully.