ECE 421 Introduction to Signal Processing

Midterm 2 – Spring 2019

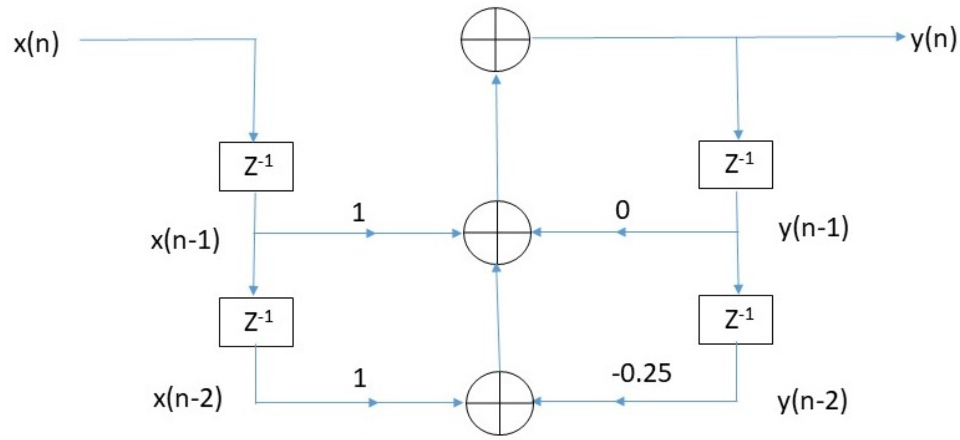
March 25, 2019

Please remember to justify your answers carefully.

There are 8 pages in total on this test.

Name: ____________________ Student ID: ____________________

Question 1 (system response)
Consider the following block diagram.



(a) Provide a difference equation that relates the input, $x(n)$, and output, $y(n)$.

(b) Compute the transfer function, $H(z) = Y(z)/X(z)$, in the z-transform domain.

(c) Based on part (b) above, sketch the pole zero plot for H. Where are the poles and zeros located? (If you are unsure about your answer in part (b), you may use the transfer function $\widetilde{H}(z) = \frac{z-1}{(z-0.5j)(z+0.5j)}$.)

(d) Based on part (c) above, what sort of filter is H? Is it a low pass, band stop, high pass, band pass, other? Make sure to justify your answer.

(e) Based on the transfer function of part (b), compute and sketch the magnitude of the frequency response, $|H(\omega)|$, as a function of $\omega \in [-\pi, +\pi]$. In your sketch, make sure to label the axes properly and highlight the effects of zeros and poles on $|H(\omega)|$. (If you are unsure about part (b), you may use $\widetilde{H}(z)$ from (c).)

Question 2 (continuous time Fourier transform)
The Fourier transform of a continuous time aperiodic signal, $x_a(t)$, is $X(F) = e^{-|F|}$, where $|F|$ denotes the absolute value of F. You will compute $x_a(t)$ in several steps.
(a) The Fourier transform $X(F)$ can be expressed as a sum,

$$X(F) = X_1(F) + X_2(F), \qquad (1)$$

where

$$X_1(F) = e^{-F}u(F), \quad X_2(F) = e^{F}u(-F),$$

and X_1 and X_2 are Fourier responses for positive and negative frequencies, respectively. Compute $x_1(t)$, the continuous time aperiodic signal that corresponds to $X_1(F)$, using the inverse transform, $x_1(t) = \int_{F=-\infty}^{+\infty} X_1(F)e^{+j2\pi Ft}dF$.

(b) Compute $x_2(t)$. (Hint: $X_2(F)$ is a reversed version of $X_1(F)$. Although we didn't discuss properties of continuous time transforms in class, you should be able to guess the relation between $x_1(t)$ and $x_2(t)$, allowing you to "double check" your calculation.)

(c) Based on linearity of the Fourier transform and equation (1) on page 4, compute $x_a(t)$ by combining your results for $x_1(t)$ and $x_2(t)$.

Question 3 (filter design considerations)
Consider a 3-tap filter of the form $h = [\underline{h(0)}, h(1), h(2)]$, where the underline corresponds to time index $n = 0$. This filter h satisfies:

- For a step function input, $x_1(n) = u(n)$, the output $y_1 = h * x_1$ at time zero satisfies $y_1(0) = 2$. (Hint: this provides information on $h(0)$.)
- The DC component of the input is magnified at the output by a factor of 3. (Hint: this provides information on $H(\omega = 0)$.)
- An input, $x_2(n) = (-1)^n$, results in an output, $y_2(n) = 5x_2(n)$. (Hint: this provides information on $H(\omega = \pi)$.)

(a) What is the impulse response, h?

(b) Compute the Fourier response, $H(\omega)$, and its magnitude, $|H(\omega)|$. (If you are unsure about your answer in part (a), you may assume $h = [\underline{3}, -2, 3]$.)

Question 4 (MATLAB)

The following MATLAB code computes the Fourier transform of a signal and plots its magnitude and phase. There are missing parts on Lines 3, 5, and 6 below.

```
1. t=0.01*(1:256); % time domain
2. m=2*cos(20*p*t); % signal
3. M=___________; % compute Fourier response
4. f=1:256; % frequencies
5. subplot(2,1,1), plot(f,___________); % plot magnitude of Fourier
6. subplot(2,1,2), plot(f,___________); % -"- phase -"-
```

(a) What is the missing code on Line 3?

(b) What is the missing code on Line 5?

(c) What is the missing code on Line 6?

Question 5 (Gibbs effect)

We learned that a discrete time aperiodic signal, $x(n)$, may have a Fourier transform $X(\omega)$ with discontinuities. In this case, $X_N(\omega)$, the transform based on truncating the signal to the range $n \in \{-N, \ldots, +N\}$ may converge to $X(\omega)$ in a mean square sense but not a pointwise sense. The lack of pointwise convergence is due to the signal $x(n)$ not being absolutely summable. Consider instead a continuous time signal, $x(t)$, that has period $T_p = 1$, and within the time interval $t \in (-0.5, +0.5)$ satisfies

$$x(t) = \begin{cases} 1 & |t| < 0.1 \\ 0 & \text{else} \end{cases}.$$

The Fourier series of $x(t)$ is $x(t) = \sum_{k=-\infty}^{+\infty} C_k e^{+j2\pi kt/T}$, and based on our derivation in class (page 7 of slides for Chapter 4) we saw that

$$C_k = 0.2\frac{\sin(0.2\pi k)}{0.2\pi k} = 0.2\text{sinc}(0.2\pi k).$$

Define a signal derived from a truncated Fourier series,

$$x_N(t) = \sum_{k=-N}^{+N} C_k e^{+j2\pi kt/T}.$$

(a) Explain why $x_N(t)$ does not converge pointwise to $x(t)$ by evaluating whether the Fourier coefficients, $(C_k)_{k=-\infty}^{+\infty}$, are absolutely summable.

(b) Nonetheless, $x_N(t)$ converges in the mean square sense to $x(t)$. Show this by using the Parseval relation to shift the discussion to mean square convergence of the Fourier coefficients, $(C_k)_{k=-\infty}^{+\infty}$, and evaluate their energy.