ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2015

February 23, 2015

Please remember to justify your answers carefully.

There are 9 pages in total on this test.

Name: _____ Student ID: _____

Question 1

Consider a system H that processes an input $x(n)$ as follows, $y(n) = 3^n x(n+2)$. We will evaluate different properties of this system.

(a) Is this a causal system? Why?

(b) Is the system bounded input bounded output (BIBO) stable or not? Why?

Question 2

Consider a sampling frequency of $F_s = 10kHz$.

(a) What is the highest range of analog frequencies that can be sampled and then perfectly reconstructed with $F_s = 10kHz$?

(b) Find two different continous-time signals that will produce the sequence

$$x(n) = \cos(0.25\pi n)$$

when sampled with $F_s = 10kHz$.

(c) Let $x_a(t) = \cos(3500\pi t) + 2\sin(12000\pi t)$. What are the discrete time frequencies of the sinusoids in the sampled sequence when $F_s = 10kHz$?

Question 3

A digital filter obeys the following difference equation,

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + \frac{7}{8}x(n).$$

The input of the filter is an impulse, i.e., $x(n) = \delta(n)$, and the initial conditions are

$$y(-1) = -1 \text{ and } y(-2) = 1.$$

We will compute the output $y(n)$ in steps using the one sided z transform.

(a) Express the one-sided z transforms of $y(n-1)$ and $y(n-2)$ using $Y^+(z)$, which is the one-sided z transform of $y(n)$, and the initial conditions.

(b) What is the one-sided transform of $x(n)$?

(c) Using your results from parts (a) and (b), express the difference equation (from the beginning of Question 3) in terms of one-sided z transforms. (Your answer should look something like $Y^+(z) = c_1 z^{-k_1} Y^+(z) + c_2 z^{-k_2} + c_3 z^{k_3}$; it will be an expression in the one-sided z domain.)
If you did not solve parts (a) and (b), you can assume that $Z^+\{y(n-1)\} = z^2 Y^+(z)$, $Z^+\{y(n-2)\} = 2z^{-2}Y^+(z)$, and $X^+(z) = 3z$.

(d) Express $Y^+(z)$ using partial fractions. (Your answer should look something like $Y^+(z) = \frac{13}{1+z^{-1}} - \frac{2}{1-4z^{-1}}$.)
If you did not solve part (c), you can assume $Y^+(z)[1 - 1.5z^{-1} + 0.5z^{-2}] = 7z^{-1}$.

(e) Compute the output $y(n)$ using the inverse one-sided z transform.
If you did not solve part (d), you can assume that $Y^+(z) = \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}} - \frac{\frac{1}{2}}{1-\frac{1}{4}z^{-1}}$.

Question 4

A linear time invariant system is described by the following difference equation,

$$y(n) = 0.5y(n-1) + bx(n),$$

where b is a constant that will be determined soon.

(a) Find the transfer function in the Fourier domain, $H(\omega) = \frac{Y(\omega)}{X(\omega)}$.

(b) What is the squared magnitude of the transfer function, $|H(\omega)|^2$?

(c) Find the value of b such that $|H(\omega)| = 1$ at frequency $\omega = 0$.
If you did not solve part (b), you can assume that $|H(\omega)|^2 = \sin(\omega) + b\cos(\omega)$.