ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2017

February 27, 2015

Please remember to justify your answers carefully.

There are 9 pages in total on this test.

Name: ______________________ Student ID: ______________________

Question 1

Consider the following system,

$$y(n) = 2x(3n - 1).$$

(a) Suppose that the input is $x(n) = \{2, -3, \underline{1}, 0, 4, -1\}$, where the underline refers to time index $n = 0$. Using the system given above, derive the output $y(n)$.

(b) Is the system time invariant? Justify your answer.

Question 2

Consider a sampling frequency of $F_s = 5kHz$.

(a) What is the highest range of analog frequencies that can be sampled and then perfectly reconstructed with $F_s = 5kHz$?

(b) Find two different continous-time signals that will produce the sequence

$$x(n) = \cos(0.2\pi n)$$

when sampled with $F_s = 5kHz$.

(c) Let $x_a(t) = \cos(2000\pi t) + 2\sin(6000\pi t)$. What are the discrete time frequencies of the sinusoids in the sampled sequence when $F_s = 5kHz$.?

Question 3

A digital filter obeys the following difference equation,

$$y(n) + 7y(n-1) + 12y(n-2) = x(n).$$

(a) What is the transfer function, $H(z) = Y(z)/X(z)$?

(b) Given that our different equation system is causal:
(i) Where are the locations of the poles and zeros?
(ii) What is the region of convergence (ROC)?
(iii) Please sketch the ROC, and include locations of its zeros and poles.
(iv) Is the system bounded input bounded output (BIBO) stable?

(c) Sketch either a Direct Form I or Direct Form II implementation of the difference equation.

Question 4

It is well known that

$$y(n) = \sum_{i=1}^{n} i = \frac{1}{2}n(n+1). \tag{1}$$

In the following steps, you will show this result using the one-sided z transform of a difference equation. To do so, we re-define $y(n)$ using the following difference equation,

$$y(n) = y(n-1) + n, \tag{2}$$

where $y(0) = 0$. (That is, the sum over the first $n = 0$ numbers is zero, because this is the sum of an empty set.) Taking the one-sided z-transform of (2),

$$Y^+(z) = [z^{-1}Y^+(z) + y(-1)] + Z^+\{nu(n)\},$$

which yields

$$Y^+(z) = \frac{y(-1) + Z^+\{nu(n)\}}{1 - z^{-1}}. \tag{3}$$

(a) Compute $y(-1)$, the value of $y(n)$ for $n = -1$. (Hint: to do so, compute $y(0)$ via (1), and then use the difference equation (2).)

(b) Compute $Y^+(z)$ using (3), the result of part (a), and the identity

$$Z^+\{nu(n)\} = \frac{z^{-1}}{(1 - z^{-1})^2}.$$

(c) Compute the one-sided z-transform of $\frac{1}{2}n(n+1)u(n)$ using the identities

$$Z^+\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^2},$$

$$Z^+\{n^2u(n)\} = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}.$$

Show that this result is equal to $Y^+(z)$ computed in (b).