ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2021

February 24, 2021

Please remember to justify your answers carefully.

Last name: ____________________ First name: ____________________

Please recall the course academic integrity policy for tests:
No cooperation or "collaboration" between students is allowed. Especially during an online course experience, it could be tempting to text or email a friend. This is not allowed. You will be allowed to use your notes, books, a browser, and software such as Matlab.[1] However, while working on the test you should not text, email, or communicate with other people (certainly not other students) in any way, unless you are consulting with the course staff. **By submitting the test, you will be acknowledging that you completed the work on your own without the help of others in any capacity.** Any such aid would be unauthorized and a violation of the academic integrity policy.

[1] You can use the browser to access Moodle, the course webpage, and look up technical topics. Similar to a normal test, you must not communicate with other people.

Question 1 (sampling)
Consider an analog signal, $x_a(t) = 0.4\cos(120\pi t + 0.6\pi) + 0.7\cos(480\pi t)$.

(a) What is the Nyquist rate required to sample $x_a(t)$?

(b) What is the maximum analog frequency that can be reconstructed by sampling an analog signal at $F_s = 100$ samples per unit time?

(c) Suppose that $x_a(t)$ is sampled at $F_s = 200$ samples per unit time. What is the resulting discrete time sampled signal $x(n)$?

(d) Was there aliasing in Part (c)? Make sure to justify your answer.

(e) The discrete time signal $x(n)$ is passed through an *ideal* digital to analog (D/A) converter using $F_s = 200$ samples per unit time. Determine the reconstructed signal in continuous time, $\widehat{x}_a(t)$.

Question 2 (difference equations and the z-transform)
Consider the following difference equation,

$$y(n) + 1.5y(n-1) = x(n) + \frac{1}{2}x(n-1).$$

(a) Compute the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

(b) Determine the zeros and poles of the system $H(z)$.

(c) Suppose that the system is causal.

- Determine the region of convergence (ROC).
- Draw the poles, zeros, and ROC.
- Is the system stable?

Question 3 (correlation)

This question will guide you through a problem where a discrete time communication signal, $x(n)$, is reflected off of some object in the environment. The receiver measures

$$y(n) = x(n) + A \cdot x(n-k),$$

where $x(n)$ is the transmitted signal, $A \cdot x(n-k)$ is the reflected signal, $|A| < 1$ is the reflection's amplitude, $k > 0$ is its delay, A is real valued, and k is integer. Our goal will be to estimate the amplitude, A, and delay, k, of the reflected signal.

(a) Recall that the autocorrelation for the transmitted signal, $x(n)$, obeys $R_{xx}(l) = \sum_n x(n)x(n+l)$. In this part, you will help us derive $R_{yy}(l)$, the autocorrelation of the version measured at the receiver. In the derivation below, α, β, and γ are missing; please specify these 3 expressions. (For your convenience, these missing parts are underlined and in red font.)

$$\begin{aligned}
R_{yy}(l) &= \sum_n y(n)y(n+l) \\
&= \sum_n [x(n) + A \cdot x(n-k)][x(n+l) + \underline{\alpha}] \\
&= \sum_n [\underline{\beta} + A \cdot x(n-k)x(n+l) + A \cdot x(n)x(n+l-k) \\
&\quad + A^2 \cdot x(n-k)x(n+l-k)] \\
&= \left[\sum_n x(n)x(n+l)\right] + A\left[\sum_n x(n-k)x(n+l)\right] \\
&\quad + A\left[\sum_n x(n)x(n+l-k)\right] + A^2 \cdot [\underline{\gamma}] \\
&= R_{xx}(l) + A \cdot R_{xx}(l+k) + A \cdot R_{xx}(l-k)] + A^2 \cdot R_{xx}(l) \\
&= R_{xx}(l)[1 + A^2] + A[R_{xx}(l+k) + R_{xx}(l-k)].
\end{aligned}$$

(b) To keep things simple, in the remainder of the question we assume that the transmitted signal, $x(n)$, has an impulse autocorrelation,

$$R_{xx}(l) = \delta(l).$$

In part (b), suppose that the delay satisfies $k = 3$. Specify $R_{yy}(l)$ as a function of A. (Hint: $R_{yy}(l)$ is nonzero for $l \in \{-k, 0, k\}$. Compute $R_{yy}(l)$ for those values of l.)

(c) Based on part (b), how would you estimate A from $R_{yy}(l)$? Keep in mind that $y(n)$ is being measured, and you must somehow estimate $R_{yy}(l)$. (If you are unsure about $R_{yy}(l)$, you may assume that $R_{yy}(-k) = A$, $R_{yy}(0) = 2A$, $R_{yy}(+k) = A^2$.)