Norms

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This supplement provides some more details about norms.

In general, the ℓ_p norm of a vector $x \in \mathbb{R}^N$ is defined as follows,

$$\|x\|_p = \left[\sum_{n=1}^{N} (x_n)^p\right]^{1/p}.$$

Taking the norm to the power of p yields

$$\|x\|_p^p = \sum_{n=1}^{N} (x_n)^p.$$

Let us discuss several different norms.

- The ℓ_0 norm is not really a norm (not strictly speaking). It is defined as

  $$\|x\|_0 = |\{n : x_n \neq 0\}|.$$

  That is, the ℓ_0 norm is the number of non-zeros in the vector. In general, the ℓ_0 norm is extremely sensitive to small values, because each of them will be counted as one.

- The ℓ_1 norm,

  $$\|x\|_1 = \sum_{n=1}^{N} |x_n|,$$

  is quite robust to outliers, because it only adds the absolute value to the summation.

- In contrast, the ℓ_2 norm is more sensitive to outliers, because

  $$\|x\|_2 = \sqrt{\sum_{n=1}^{N} (x_n)^2},$$

  and one outlier can easily dominate the summation.

- The ℓ_∞ norm is most sensitive to outliers,

  $$\|x\|_\infty = \max_n |x_n|.$$