

# Direct Sums

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This supplement provides more details about direct sums.

**Direct sums:** Consider two vector spaces,  $V_a$  and  $V_b$ , and two corresponding signals,  $x_a \in V_a$  and  $x_b \in V_b$ . A direct sum, denoted by  $V_c = V_a \oplus V_b$ , is the union of all sums of the form  $x_a + x_b$ . That is,

$$V_a \oplus V_b = \{x : \exists x_a \in V_a, x_b \in V_b, x = x_a + x_b\}.$$

As a simple example, consider  $V_a$  to be the set of all signals band limited to  $\omega_1$ , and  $V_b$  the set of all signals band limited between  $\omega_1$  and  $\omega_2 > \omega_1$ . The direct sum is the set of all signals band limited to  $\omega_2$ .