# LASSO

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This supplement provides more details about the LASSO algorithm.

**LASSO:** The LASSO optimizes the following formulation,

$$\widehat{x} = \arg\min_{x \in \mathbb{R}^N} \|x\|_1 + \lambda \|y - \Phi\|_2^2,$$

where $\lambda$ is a Lagrangian parameter, which accounts for the trade-off between the sparsity level of the signal $x$ and $y - \Phi x$, the unexplained portion of the measurements.

Let us focus on two extremes for $\lambda$. First, take $\lambda \to \infty$. In this case, the optimization enforces $y - \Phi x = 0$, else the second term becomes infinitely large. Therefore, the optimization becomes

$$\widehat{x} = \arg\min_{x s.t. y = \Phi x} \|x\|_1,$$

which is identical to $\ell_1$ optimization. Second, take $\lambda \to 0$. This means that the unexplained portion of the measurements becomes unimportant, and the algorithm minimizes $\|x\|_1$. But $x = 0$ has zero $\ell_1$ norm, and so the limit of the LASSO solution tends to zero,

$$\lim_{\lambda \to 0} \widehat{x} = 0.$$

Medium values of the Lagrangian $\lambda$ provide a more insightful balance between the unexplained portion of the measurements, which can correspond to measurement noise or small nonzeros in the signal, and the sparsity of the solution.