

1. What is the rank of the matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ ? Why?
2. Provide an expression for  $x \in \mathbb{R}^2$  in terms of  $A \in \mathbb{R}^{2 \times 2}$  and  $b \in \mathbb{R}^2$  given that  $AX = X + b$
3. Assume that  $x$  is a random variable with probability distribution  $p(x)$ ; provide an expression for the expected value of  $x^2$
4. Consider two events,  $E_1$  and  $E_2$  with probabilities  $p_1=0.5$  and  $p_2=0.4$ , and suppose further that  $\Pr(E_1, E_2)=0.1$ . Are the events independent? Why?

# Answer

1. It is 2, because the first two rows are clearly independent but the last one is the sum of the first two.
2. We can rewrite the expression as  $(A - I)X = b$ , where  $I$  is the identity matrix. Then, assuming that  $(A - I)$  is invertible, we get  $X = (A - I)^{-1}b$
3. We have  $E[x] = \sum_x x^2 \cdot p(x)$
4. If they were independent, then the joint probability is the product of the two individual probabilities. But  $0.5 \times 0.4 = 0.2$ , which differs from 0.1.