ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 1

Dror Baron; Spring 2023
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Administrative instructions:

1. The homework must be submitted individually.
2. You should submit in class the day that the homework (HW) is due (hard copy), or electronically by midnight that day. How about we just do hard copy in a small course?
3. Handwritten answers are fine.
4. Please justify your answers carefully.

Question 1
Prove Euler's identity,

$$\exp(i\theta) = \cos(\theta) + \sin(\theta)i.$$

To do so, use Taylor series expansions for $f_1(x) = \exp(x)$, $f_2(x) = \sin(x)$, and $f_3(x) = \cos(x)$. You may use the results we derived in class for f_1 and f_2, although if you are rusty with this material, working through the entire question will be advantageous.

Question 2
Recall our formula for an infinite geometric series,

$$S(x) = \sum_{n=0}^{+\infty} x^n = \frac{1}{1-x},$$

provided that $|x| < 1$. In class, we derived this result by comparing $S(x)$ and $xS(x)$. Please derive the Taylor series for $\frac{1}{1-x}$ and confirm that it coincides with $\sum_{n=0}^{+\infty} x^n$. Why would this derivation fall apart for $|x| > 1$?

Question 3
Now that you are proficient with Euler's identity, let's use it to demonstrate some relations in trigonometry.

- Show that $\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$. Hint: in addition to Euler, you can usethe following symmetry relations, $\sin(\theta) = -\sin(-\theta)$ and $\cos(\theta) = \cos(-\theta)$.
- Show that $\sin(\theta) = \frac{1}{2j}(e^{i\theta} - e^{-i\theta})$.
- Show that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$. Hint: you may spot an expression of the form $(a+b)(a-b)$, which equals $a^2 - b^2$, in your derivation.