ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 2

Dror Baron; Spring 2023
February 8, 2023

Administrative instructions:

1. The homework must be submitted individually.
2. You should submit in class the day that the homework (HW) is due (hard copy), or electronically by midnight that day.
3. Handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (matrix composition; page 64 in Nielsen and Chuang.)
Suppose that V, W, and X are linear vector spaces, and $A : V \to W$ and $B : W \to X$ are linear operators. We use the notation BA to denote the composition of A and B, meaning that $(BA)\,|\psi\rangle = B(A(|\psi\rangle))$. Show that the matrix representation of the BA linear operator is the product of the 2 matrices, A and B, that correspond to these linear operators. (To do so, you may want to define bases for V, X, and W.)

Question 2
In class we considered the following matrix,

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}.$$

Express this matrix in the $|+\rangle$ and $|-\rangle$ basis. That is, if the coordinates of some vector $|\psi\rangle$ are given with respect to (w.r.t.) the $|+\rangle$ and $|-\rangle$ basis, i.e., $|\psi\rangle = \alpha\,|+\rangle + \beta\,|-\rangle$, then find γ and δ such that $A\,|\psi\rangle = \gamma\,|+\rangle + \delta\,|-\rangle$, where γ and δ should be linear functions (meaning matrix multiplication) of α and β.

Question 3
Recall the Cauchy-Shwartz inequality, $|\,\langle v|w\rangle\,|^2 \leq \langle v|v\rangle\,\langle w|w\rangle$. When does equality hold?

Question 4
In class, we saw that the eigen-values of the Pauli-X matrix are ± 1. Compute the eigen-values and eigen-vectors for one of the other "standard" matrices, Y, Z, or H (Hadamard).

Question 5 (Exercise 2.24 in Nielsen and Chuang.)
Show that a positive operator is Hermitian. (Hint: show that any linear operator A can be written as $A = B + iC$ where B and C are both Hermitian.)