ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 4

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Administrative instructions:

1. The homework must be submitted individually.
2. You should submit in class the day that the homework (HW) is due (hard copy), or electronically by midnight that day.
3. Handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (Sampling a signal.)
Consider taking a discrete time signal $x(n)$ and evaluating only even numbered samples, $\widetilde{x}(n) = x(2n)$. We can compute

$$\widetilde{X}(\omega) = \sum_{n=-\infty}^{+\infty} x(2n)e^{-in\omega}$$

using values of $x(n)$. In this question, we will express $\widetilde{X}(\omega)$ using $X(\omega)$.
(a) Define $x_a(n) = x(n)(-1)^n$. Express $X_a(\omega)$ in terms of $X(\omega)$ without using the actual sample values, $x(n)$. (Hint: consider the form $X_a(\omega) = c_1 X(c_2\omega + c_3)$ for some constants.)
(b) Define $x_b(n) = x(n) + x_a(n)$. How does $x_b(n)$ relate to $\widetilde{x}(n)$?
(c) Express $\widetilde{X}(\omega)$ using the above results.

Question 2 (Low pass filters.)
The following signals can all be interpreted as low pass filters, meaning that they attenuate higher frequencies while letting lower ones pass through.
(a) Consider an averaging filter with $2M+1$ taps. That is, $h(n) = \frac{1}{2M+1}$ when $|n| \leq M$, else $h(n) = 0$. What is $H(\omega)$? (Note that the value $\frac{1}{2M+1}$ ensures that $|H(0)| = 1$.)
(b) Consider a triangular filter defined over the same range, $|n| \leq M$. Samples $h(n)$ have the form $h(n) = (M+1-|n|)/A$ for $n \in \{-M, -M+1, \ldots, M\}$. Note that $|h(0)|$ is largest. Design A in a way that the filter's DC response is 1, meaning that it barely impacts low frequencies.
Programming component. Compare the frequency response, $|H(\omega)|$, for these 2 filters. Discuss how the frequency responses compare.

Question 3 (Measurement operators.)
Recall that $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$ are projection matrices. Consider using the following set of measurement operators, M_0, $0.8M_1$, and $0.6M_1$.
(a) Show that the completeness relation holds, i.e., $\sum_m M_m^\dagger M_m = I$.
(b) For $|\psi\rangle = [\alpha\ \beta]^T$, compute the probabilities of measuring M_0, $0.8M_1$, and $0.6M_1$.

Question 4 (Observables.)
Consider the observable matrix, $M = 2|+\rangle\langle +| - |-\rangle\langle -|$.
(a) Is M is Hermitian? Unitary? Normal? Positive?
(b) What is the average measurement if $|\psi\rangle$ is $|0\rangle$?