ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 4

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Administrative instructions:

1. The homework must be submitted individually.
2. You should submit in class the day that the homework (HW) is due (hard copy), or electronically by midnight that day.
3. Handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (Parallel quantum operators.)
Consider 2 qubits, $|\psi_1\rangle$ and $|\psi_2\rangle$. We now apply X to the first qubit and Z to the second qubit. These operations can be performed in parallel, meaning that the following systems have the same outputs: (i) form $|\psi_3\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, then apply X to get $|\psi_4\rangle$, then apply Z to get $|\psi_5\rangle$; (ii) similar to above, but swap the order of X and Z; and (iii) apply X and Z together, meaning that we go from $|\psi_3\rangle$ directly to $|\psi_5\rangle$. Your role is to show that $|\psi_5\rangle$ is identical using all 3 approaches.

Question 2 (Controlled Z gate.)
(This is exercise 4.18 in Nielsen & Chuang.) Show that applying a controlled Z operation to the first qubit is identical to applying a controlled Z to the second one. To make things more concrete, applying a controlled Z to the second qubit means that if the control qubit (the first one) is 1, then we apply Z to the second target qubit, else the second qubit is unchanged (we apply I). Applying a controlled Z to the first qubit means that the roles of the qubits are changed.

Question 3 (H_3 Hadamard matrix.)
In class, we derived H_1, the Hadamard matrix for 1 qubit (a 2×2 matrix) and H_2 (4×4). Derive H_3, the 8×8 Hadamard matrix.

Question 4 (The Phineas challenge.)
Prove that $H^{\otimes n}$, the Hadamard transform applied to n qubits, can be implemented by applying the "standard" Hadamard ($H = H_1$) to each of the n qubits in parallel. To do so, use induction on n.

Question 5 (Bernstein-Vazirani example.)
In class, we considered an example Bernstein-Vazirani problem where we had $n = 2$ qubits, the hidden string was $s = 11$, the output of the second Hadamard was $|\psi_4\rangle = \frac{1}{2}\left[|{+}{+}\rangle - |{+}{-}\rangle - |{-}{+}\rangle + |{-}{-}\rangle\right]$, and we showed that the quantum amplitudes of 00 and 11 were 0 and 1, respectively.
(a) Show that the quantum amplitudes of the remaining computational basis states, 01 and 10, are both 0.
(b) Take a hidden string $s = 01$, recompute the output of the $Q_F^{\pm}$ circuit, $|\psi_3\rangle$, recompute the output of the second Hadamard, $|\psi_4\rangle$, and show what is measured in the computational basis state.