ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 6

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Administrative instructions:

1. The homework must be submitted individually.
2. You should submit in class the day that the homework (HW) is due (hard copy), or electronically by midnight that day.
3. Handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (Simon's problem.)
Your goal in this problem will be to describe an example for Simon's problem.
(a) Define m and n. (Use $n \geq 3$, else this question becomes too easy.)
(b) Specify L, the unknown period.
(c) Specify a table comprised of inputs and outputs of the function F for all 2^n possible inputs.
(d) Sample from the quantum circuit. Specifically, provide $n-1$ values of $s \in \{0,1\}^n$ such that $\text{mod}\left(\langle s, L\rangle, 2\right) = 0$.
(e) Form a linear program that expresses a matrix of s vectors that multiplies the unknown L. Show that the $n-1$ vectors are linearly independent. (Hint: if by bad luck your $n-1$ vectors are linearly dependent, try to design a set of $n-1$ linearly independent vectors. To do so, you can sample more and more vectors until you have $n-1$ linearly independent ones.)
(f) Solve the linear problem.

Question 2 (FFT.)
Implement the FFT as we discussed in class. Your FFT need only run for $N = 2^n$. Irrespective of what programming environment you use, make sure to compare your implementation to the standard one to confirm that your FFT returns the same answer (floating point precision issues are fine). Finally, plot the runtimes for the DFT and FFT for several problem sizes, where you will likely stop the DFT around $n = 15$.

Question 3 (Quantum phase estimation.)
Describe a matrix U that can be used in quantum phase estimation. Make sure that U is unitary, and that its phase ϕ is "interesting." Specify ϕ and $|u\rangle$.