Two-Part Reconstruction in Noisy Compressed Sensing

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Motivation

Noisy compressed sensing problem

- Input signal: $x \in \mathbb{R}^N$
- Measurement matrix: $\Phi \in \mathbb{R}^{M \times N}$, often $M < N$
- Measurements: $y = \Phi x + z$, $z \in \mathbb{R}^M$ is noise
- Goal: reconstruct x given Φ and y

Fast CS reconstruction algorithms

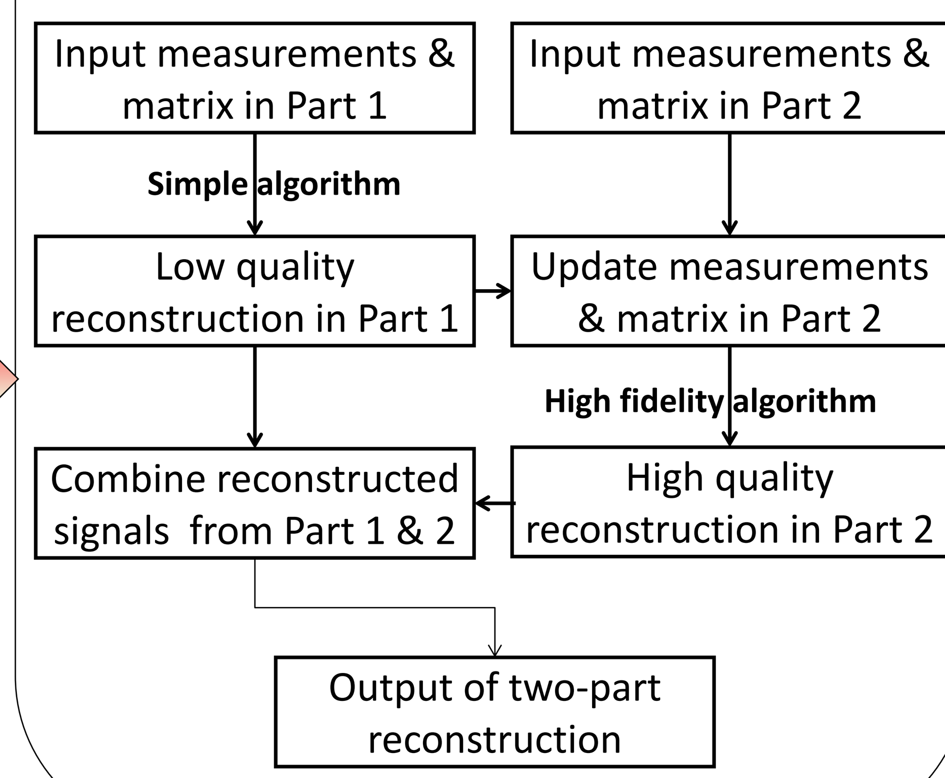
☺ Simple and fast ☹ Lower fidelity

High fidelity CS reconstruction algorithms

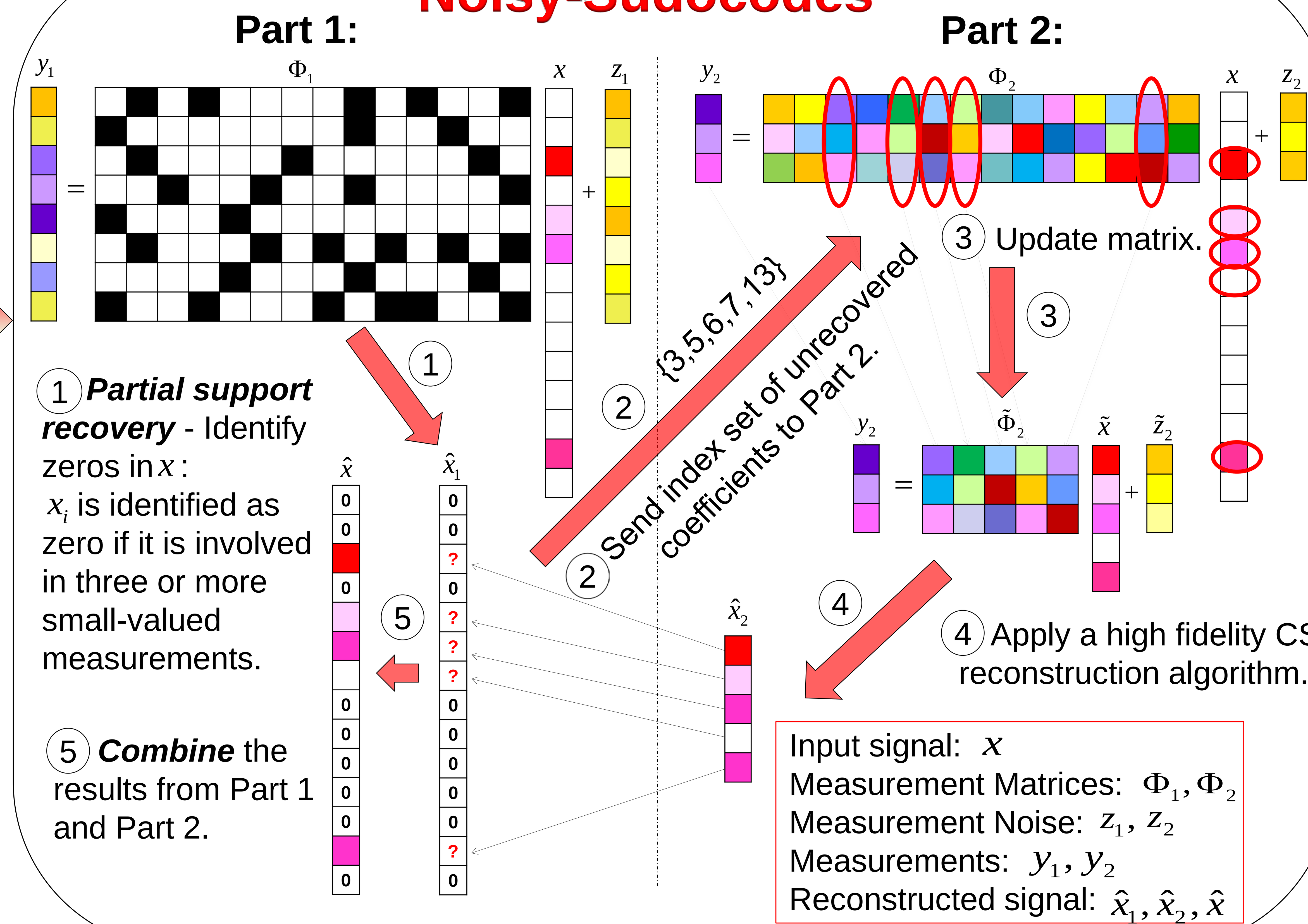
☺ Resilient to noise ☹ Complex and slow

What about combining the advantages of different algorithms?

Two-Part Reconstruction



Noisy-Sudocodes

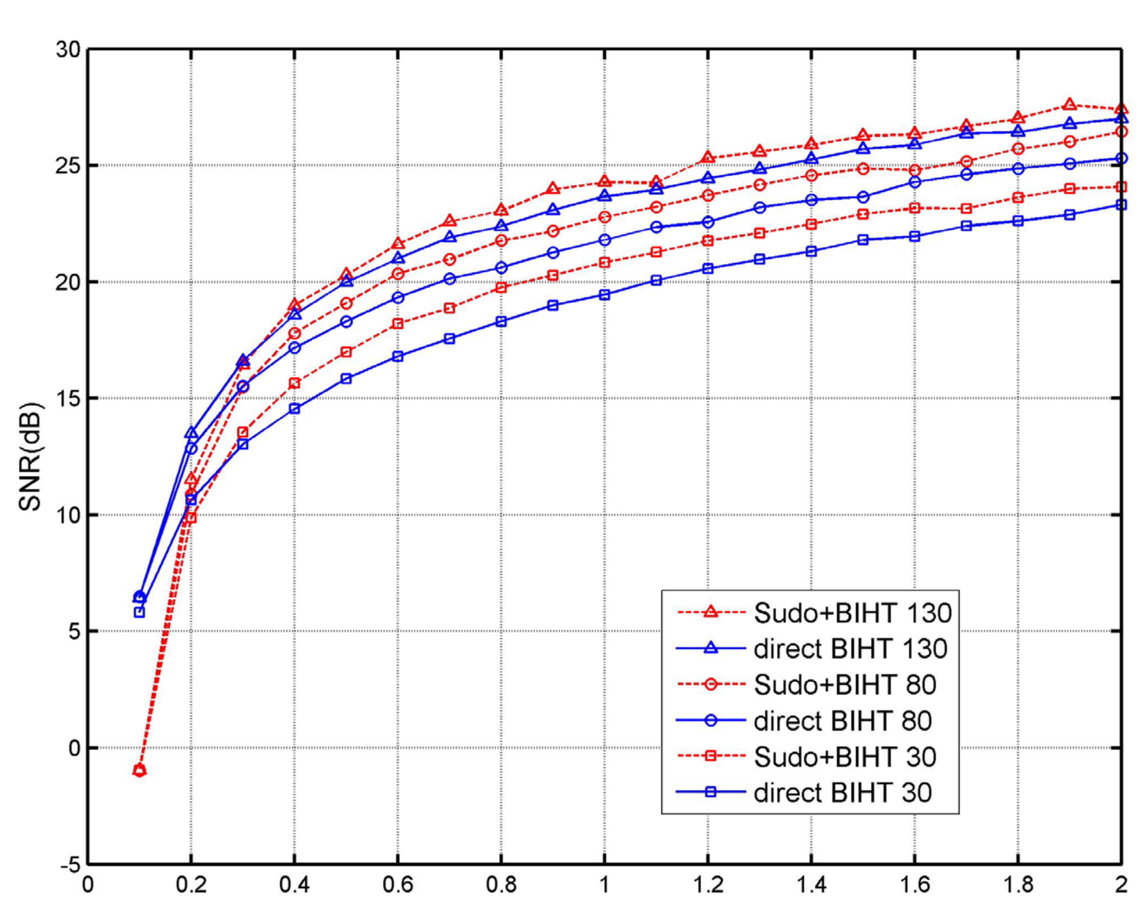


Input signal: x
Measurement Matrices: Φ_1, Φ_2
Measurement Noise: z_1, z_2
Measurements: y_1, y_2
Reconstructed signal: $\hat{x}_1, \hat{x}_2, \hat{x}$

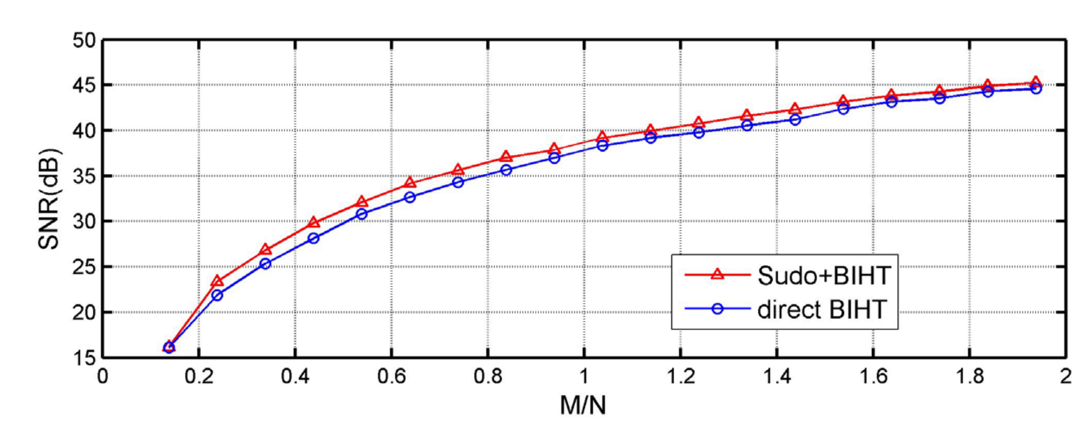
Numerical Results

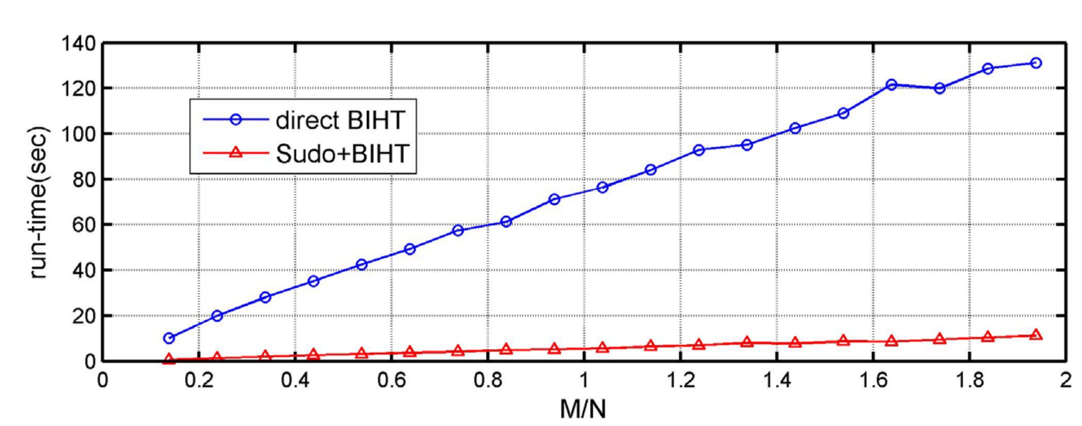
Experimental setting:

- Compare Sudo+BIHT to original BIHT.
- Sparse Gaussian input x with $\|x\|_2 = 1$; input length N=10,000; sparsity K=50.
- i.i.d. Gaussian noise $z \sim N(0, \sigma_z^2)$ in noisy setting, where $\sigma_z^2 = 10^{-2.5}$.



SNR achieved by Sudo+BIHT and original BIHT with 30, 80 and 130 BIHT-iterations in noisy 1-bit CS setting





SNR & run-time achieved by Sudo+BIHT and original BIHT in noiseless 1-bit CS setting

Note: Part 1 of Noisy-Sudocodes does not introduce error in noiseless setting.

Conclusion: Noisy-Sudocodes achieve

- Improved SNR
- Reduced run-time

Application to 1-bit CS

Conventional 1-bit quantizer

Noiseless 1-bit CS: $\bar{y} = \mathrm{sign}(\Phi x)$ - quantization noise

Noisy 1-bit CS: $\bar{y} = \mathrm{sign}(\Phi x + z)$ - quantization noise & additive measurement noise

New 1-bit quantizer

$$\bar{y}_j = \begin{cases} 0, & |\langle \Phi_j, x \rangle + z_j| \le \varepsilon \\ 1, & |\langle \Phi_j, x \rangle + z_j| > \varepsilon \end{cases}$$

$\Phi \in \{0,1\}^{M \times N}$, $j = \{1, \cdots, M\}$, Φ_j : the jth row of Φ,
$\varepsilon \ge 0$ ($\varepsilon = 0, z = 0$ for noiseless setting).

Noisy-Sudocodes applied to 1-bit CS (Sudo+BIHT)

Part 1: utilize the new 1-bit quantizer;
Part 2: utilize existing 1-bit CS algorithm: Binary Iterative Hard Thresholding (BIHT).
BIHT-$l1$ for noiseless setting; BIHT-$l2$ for noisy setting.