Compressive Imaging via Approximate Message Passing with Wavelet-Based Image Denoising

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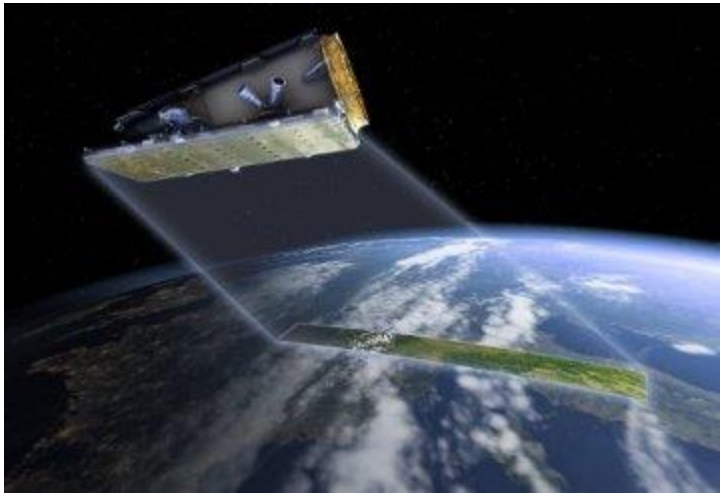
Compressive Imaging

Compressive imaging

Less radiation

Less power consumption



Compressive imaging

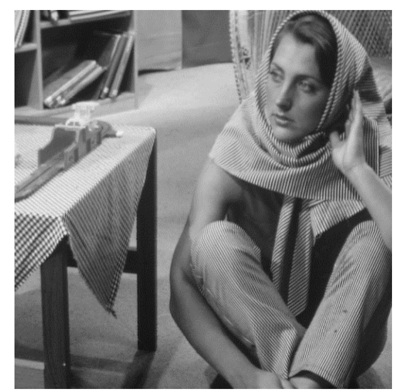
Dimension N

$N \gg M$

Dimension M

Linear measurements

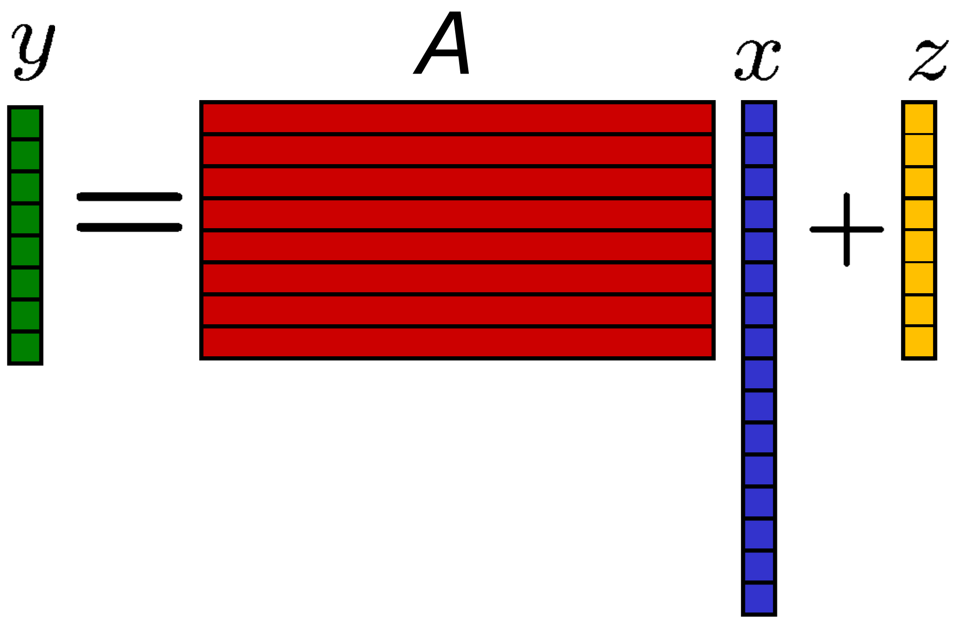
Dimension M

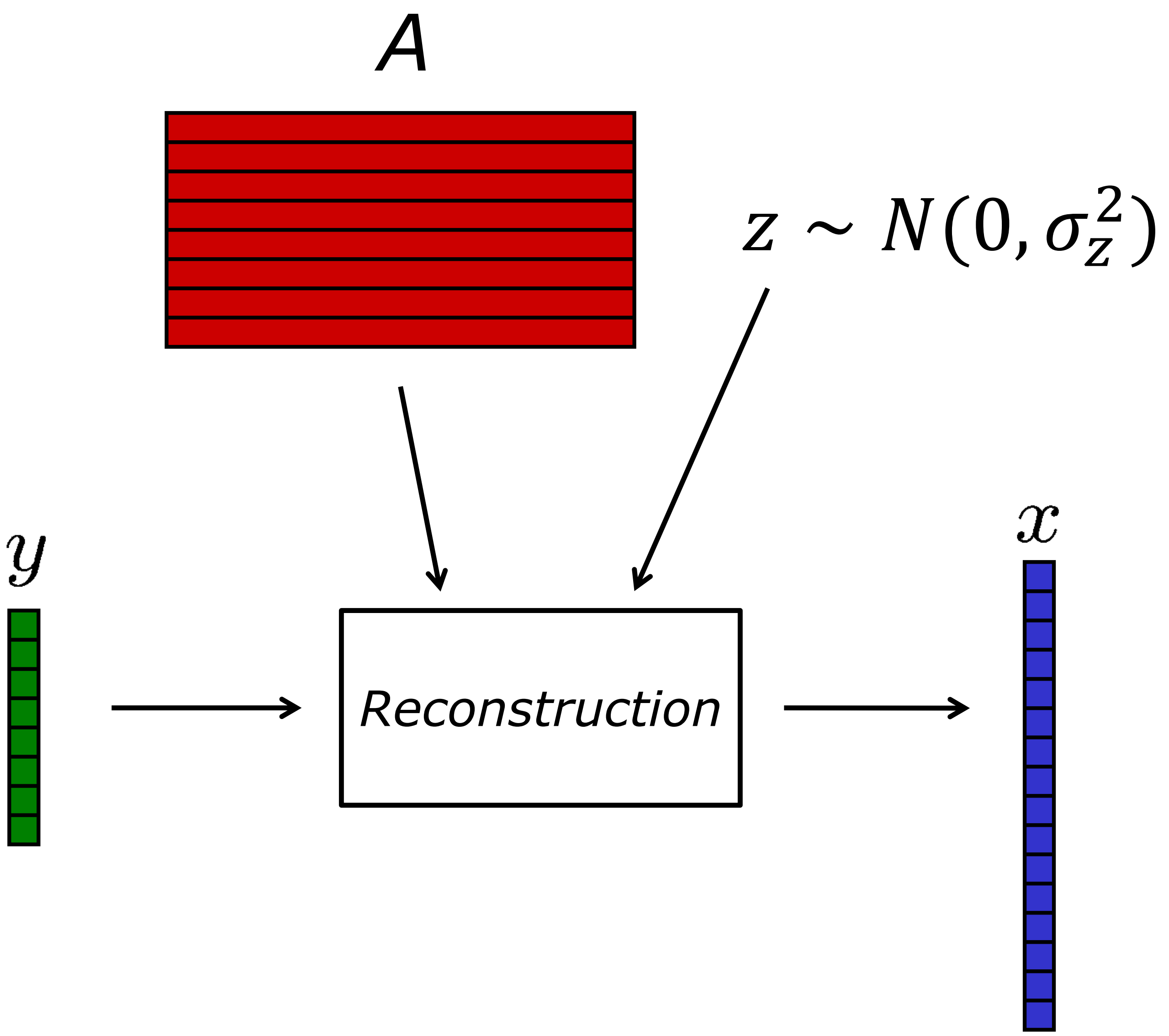


Dimension N

Compressive imaging

- Length-N input x
- Matrix A, dimension M*N, M<N
- Additive white Gaussian noise
- Well known: modest # measurements M suffices for robust signal reconstruction





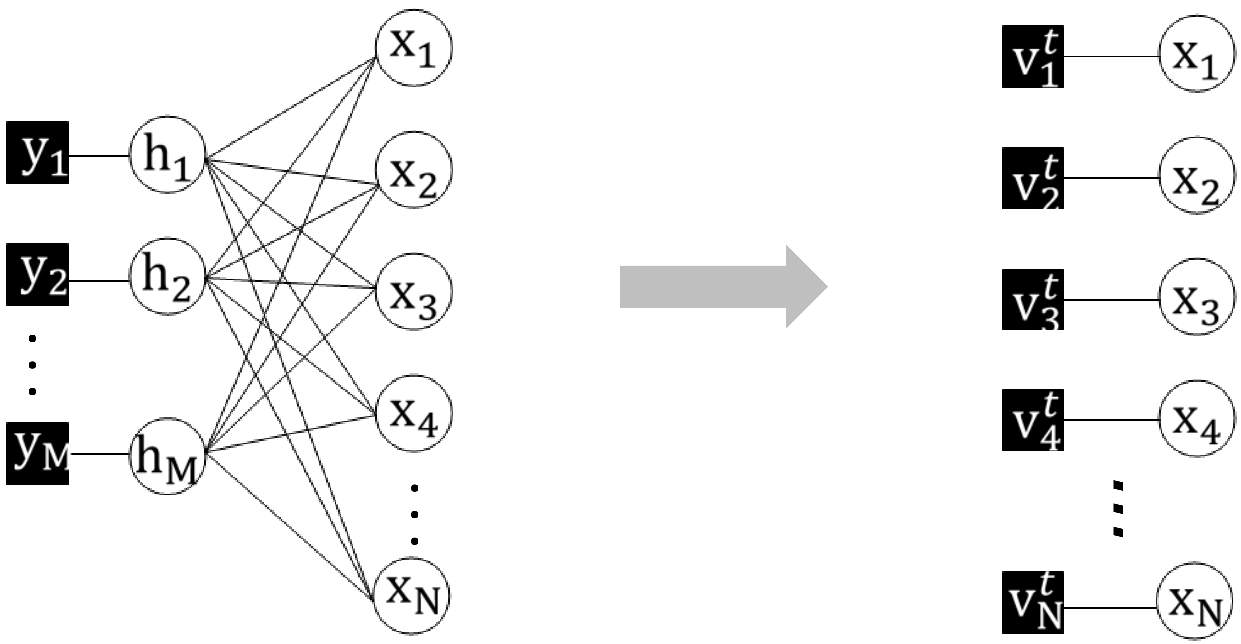
A
z ~ N(0, σ_z^2)
y
Reconstruction
x

Approximate Message Passing

Approximate message passing (AMP)

[Donoho, Maleki, & Montanari 2009]

- Fast iterative algorithm
- Based on belief propagation



$$y = h + z = Ax + z \qquad v = x + noise$$

Approximate message passing (AMP)

Initialize $x^0 = 0$

At iteration t, do

Residual: $$r^t = y - Ax^t + \frac{r^{t-1}}{M/N}\langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1})\rangle$$

Noisy image: $$v^t = x^t + A^T r^t$$

Denoising: $$x^{t+1} = \eta_t(v^t)$$

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Onsager term

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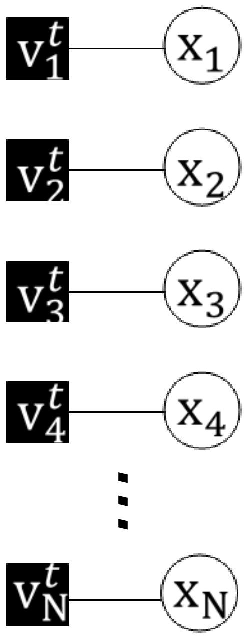


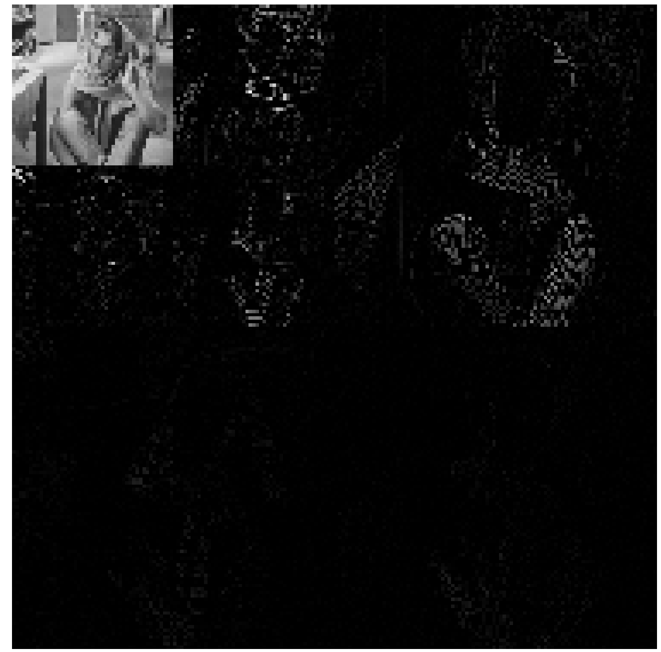
Standard AMP: $\eta_t(v^t)$ is scalar

Wavelet-Based Image Denoising

Wavelet-based image denoising

Wavelet-based, convenient for Onsager term computation

Original image

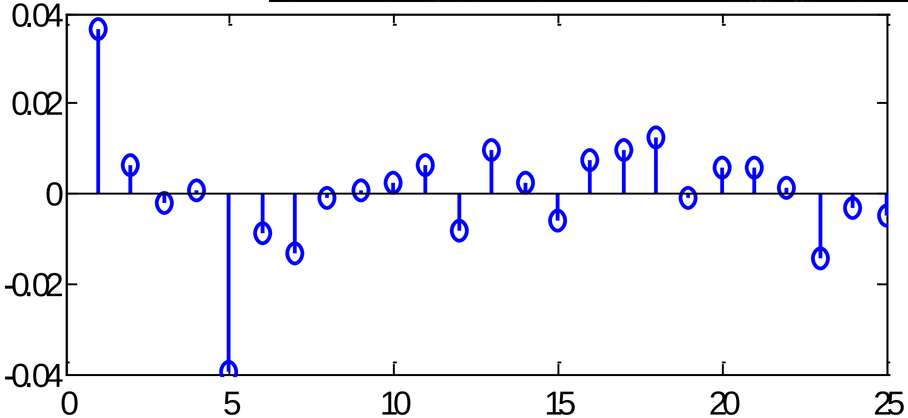


Wavelet transform

Wavelet-based image denoising

Adaptive Wiener filter [Mihcak et al. 1999]

Values of neighboring coefficients



Wavelet-based image denoising

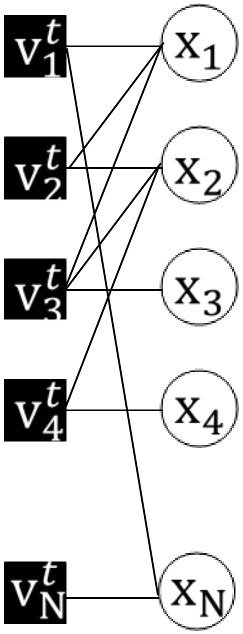
Adaptive Wiener filter [Mihcak et al. 1999]

Compute variances of wavelet coefficients based on neighborhood, σ_i^2

Adaptive Wiener filtering: $\frac{\sigma_i^2}{\sigma_i^2+\sigma_z^2}$ $\cdot$*i-th noisy wavelet coefficient*

Wavelet-based image denoising

Adaptive Wiener filter [Mihcak et al. 1999]



Non-scalar denoiser

Compute variances of wavelet coefficients based on neighborhood, σ_i^2

Adaptive Wiener filtering: $\dfrac{\sigma_i^2}{\sigma_i^2+\sigma_z^2}$ ·*i-th noisy wavelet coefficient*

AMP with Adaptive Wiener Filter

(AMP-Wiener)

AMP-Wiener

Initialize $x^0 = 0$

At iteration t, do

Residual: $r^t = y - Ax^t + \frac{r^{t-1}}{M/N}\langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1})\rangle$

Noisy image: $v^t = x^t + A^T r^t$

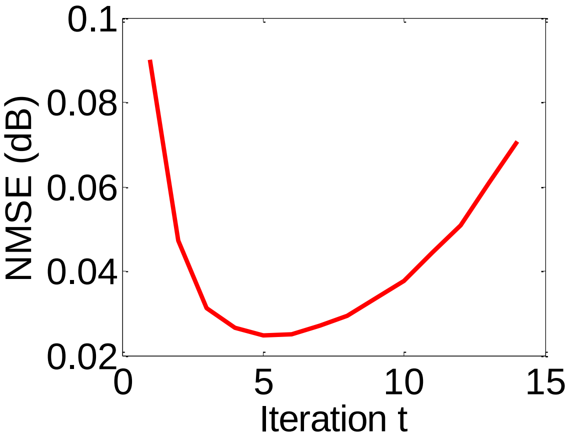
Denoising: $x^{t+1} = \eta_t(v^t)$

AMP-Wiener

Denoising: $x^{t+1} = \eta_t(v^t)$

✓ Noise variance is approx. $||r^t||_2^2/M$ [Montanari 2012]

✓ Divergence problem



AMP-Wiener

Denoising: $x^{t+1} = \eta_t(v^t)$

✓ Noise variance is approx. $||r^t||_2^2/M$ [Montanari 2012]

✓ Divergence problem: damping [Rangan et al. 2014]
$\alpha \cdot$ current estimate + $(1-\alpha) \cdot$ previous estimate
$(0 < \alpha \leq 1)$

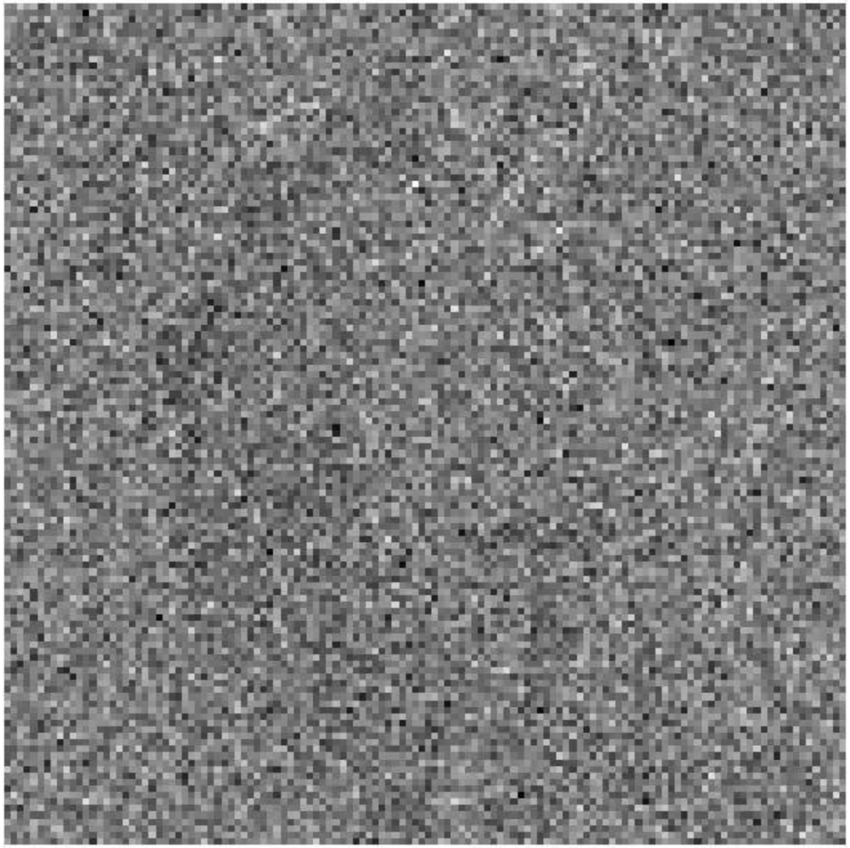
Numerical Results

Numerical results

Original

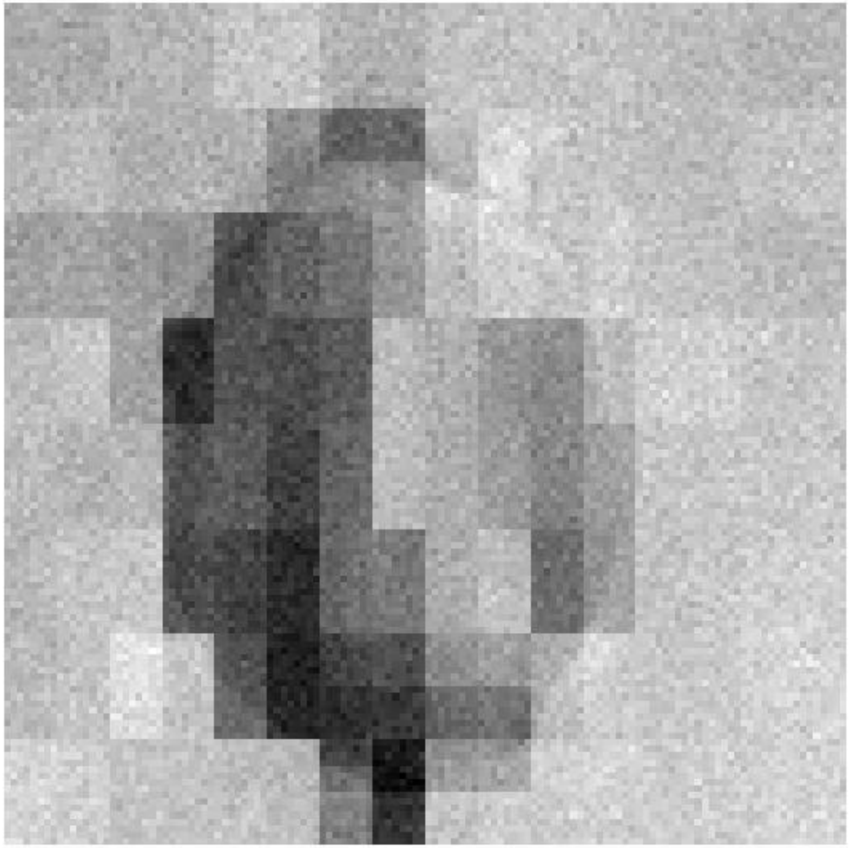
Numerical results

Iteration 1



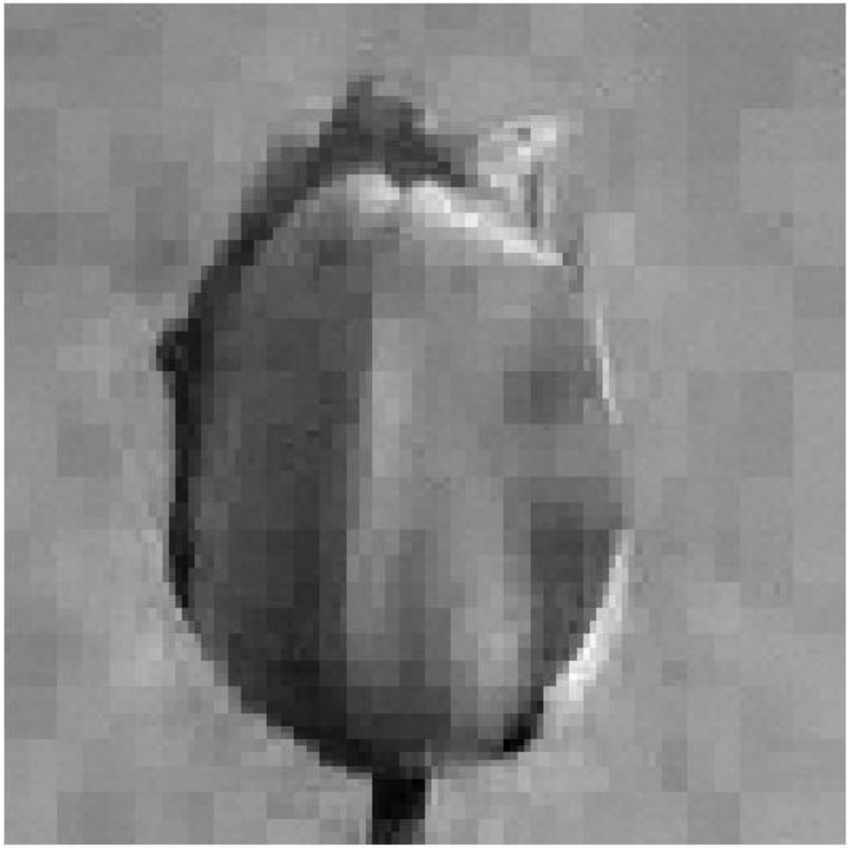
Numerical results

Iteration 3



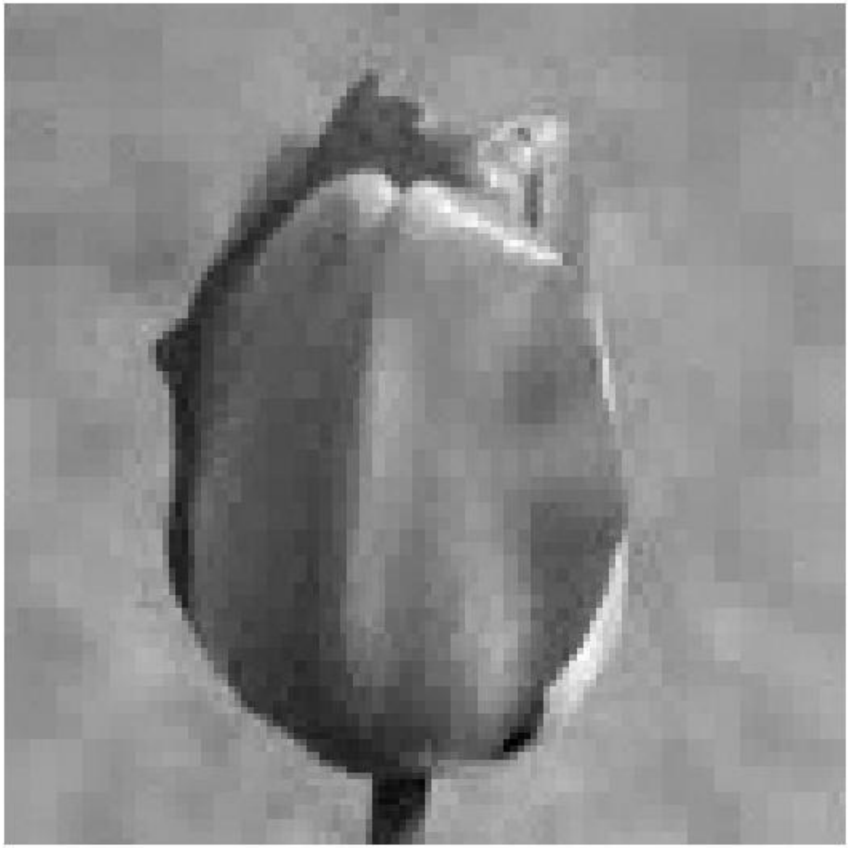
Numerical results

Iteration 7



Numerical results

Iteration 30



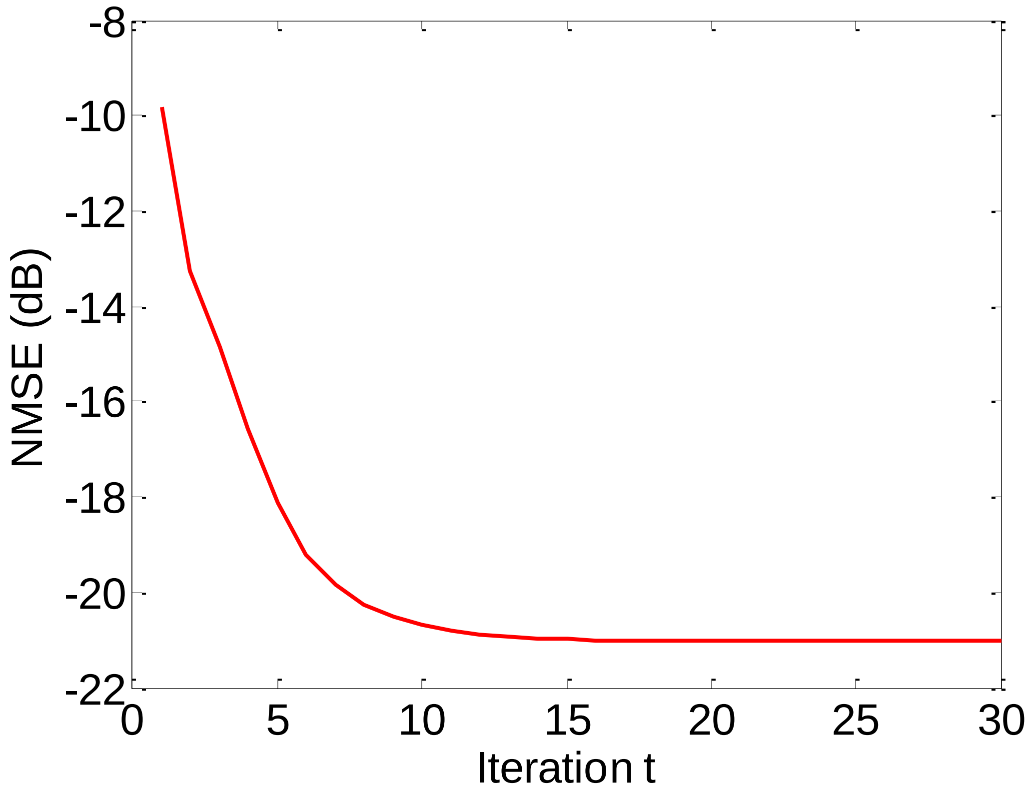
Numerical results

A: i.i.d. zero-mean Gaussian, Measurement rate 0.3
Average over 591 images

Algorithm	NMSE(dB)	Runtime(sec)
Turbo-BG	-20.37	12.39
Turbo-GM	-20.72	12.47
MCMC	-20.31	>400
AMP-Wiener	**-21.00**	**3.34**

[Turbo-BG/GM: Som & Schniter 2012]
[MCMC: He & Carin 2009]

Numerical results



Numerical results

AMP-Wiener [Tan et al., May 2014]

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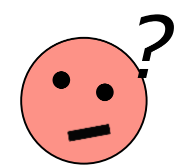
$$r^t = y - Ax^t + \frac{r^{t-1}}{M/N}\langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1})\rangle$$



$$= \eta_t \left(\quad \right)$$

Numerical results

AMP-Wiener [Tan et al., May 2014]

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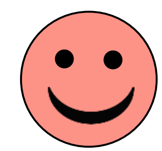

AMP-BM3D with Monte Carlo
[Metzler et al., June 2014]

Numerical results

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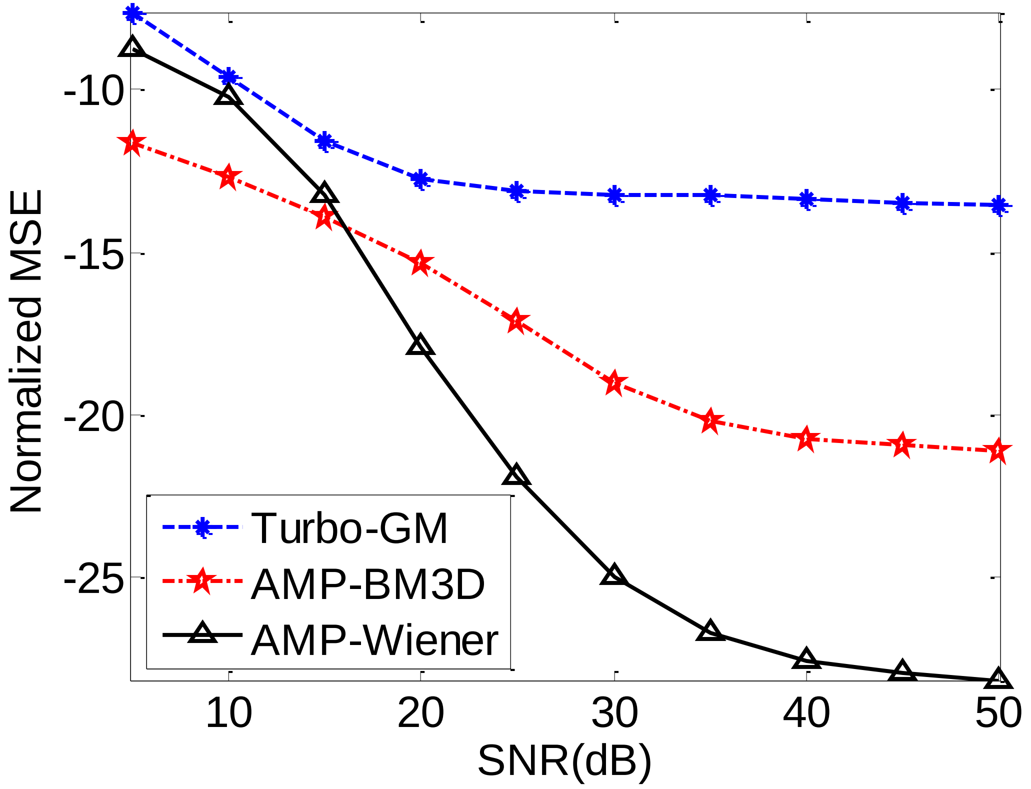
Turbo-BG	-20.37	12.39
Turbo-GM	-20.72	12.47
MCMC	-20.31	>400
AMP-Wiener	-21.00	**3.34**
AMP-BM3D	**-25.27**	16.06

$\boldsymbol{A}$: *i.i.d. zero mean Gaussian*

Radio astronomy imaging system

$$y = \text{blurring kernel} * x + z$$

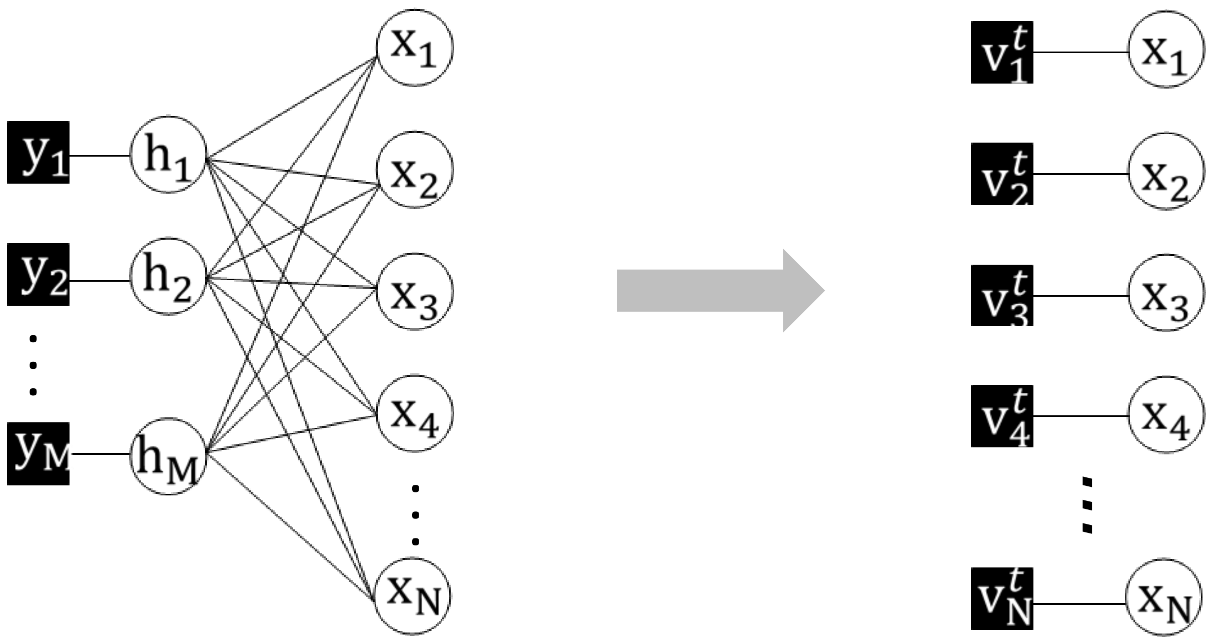


Conclusion

Conclusion

Approximate message passing:

Convert matrix channel problem to scalar denoising problem



Conclusion

Denoising via adaptive Wiener filter:

$$x^{t+1} = \eta_t(v^t)$$

Conclusion

Adaptive Wiener filter:
A robust denoiser with simple derivative

$$r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1}) \rangle$$

Onsager term

Questions?