Fault Identification and Compressed Sensing

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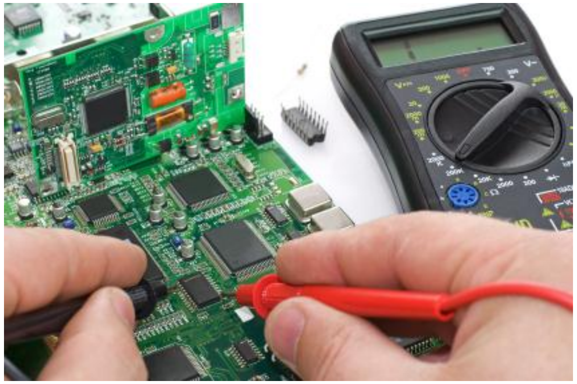
Motivation

- *What's the problem?*
  - pattern of faults (errors / breakages)
  - need to determine pattern

- *Applications*
  - aerospace
  - automotive systems
  - industrial process control
  - analog circuit example derived in detail [Bickson et al. 2011]



Measuring Faults

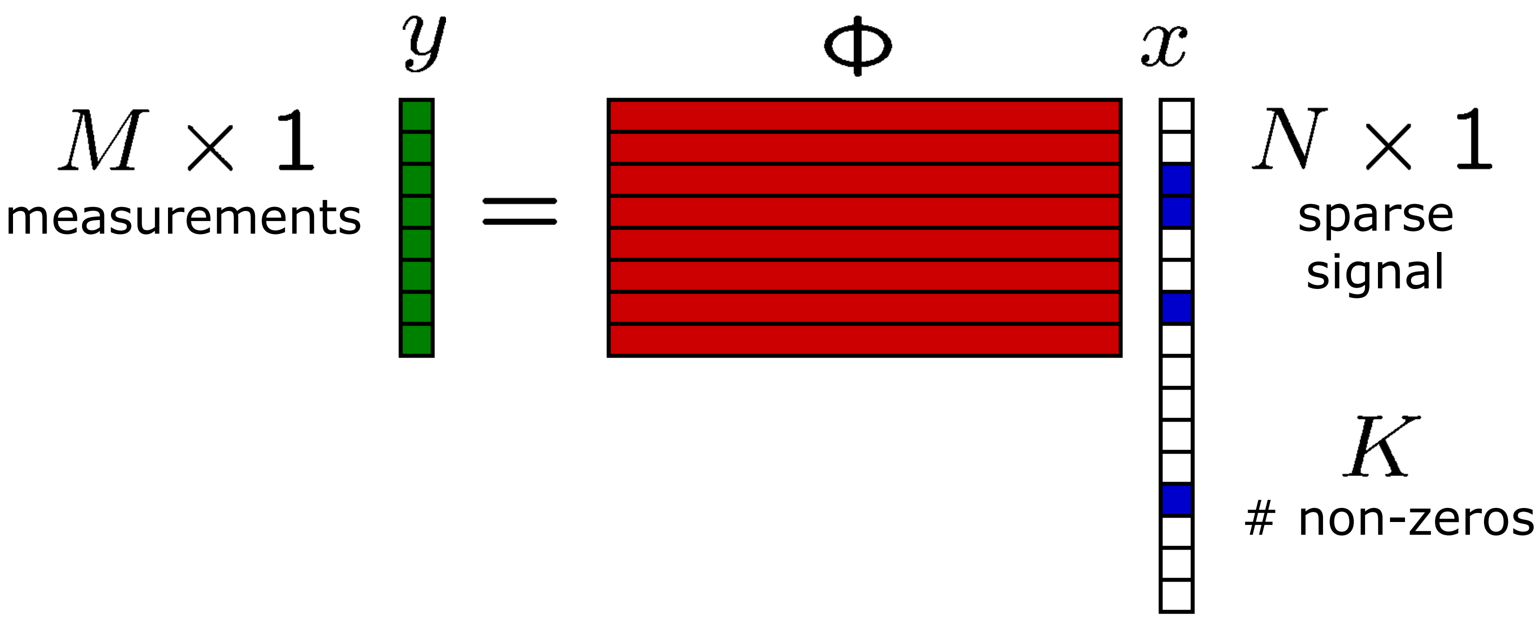
- Acquire vector of measurements
- Each possible faults has *signature*
  (typically sparse)
- Each signature *could* appear in measurement vector
  (few faults → most don’t appear)

$\Rightarrow$ *Superposition of small number of sparse signatures*

Fault Identification Model

- Signal entry $B_n \sim$ *Bernoulli*(p) (coin flips)
- Random iid measurement matrix, Pr(nonzero)=q
- Length-N signal; M measurements
- Large system limit: $\lim_{N\to\infty} \frac{M}{N} = \delta > 0$

$$y = \Phi x$$



Measurement Noise

- Measurement process is typically analog
- Analog systems add noise, non-linearities, etc.

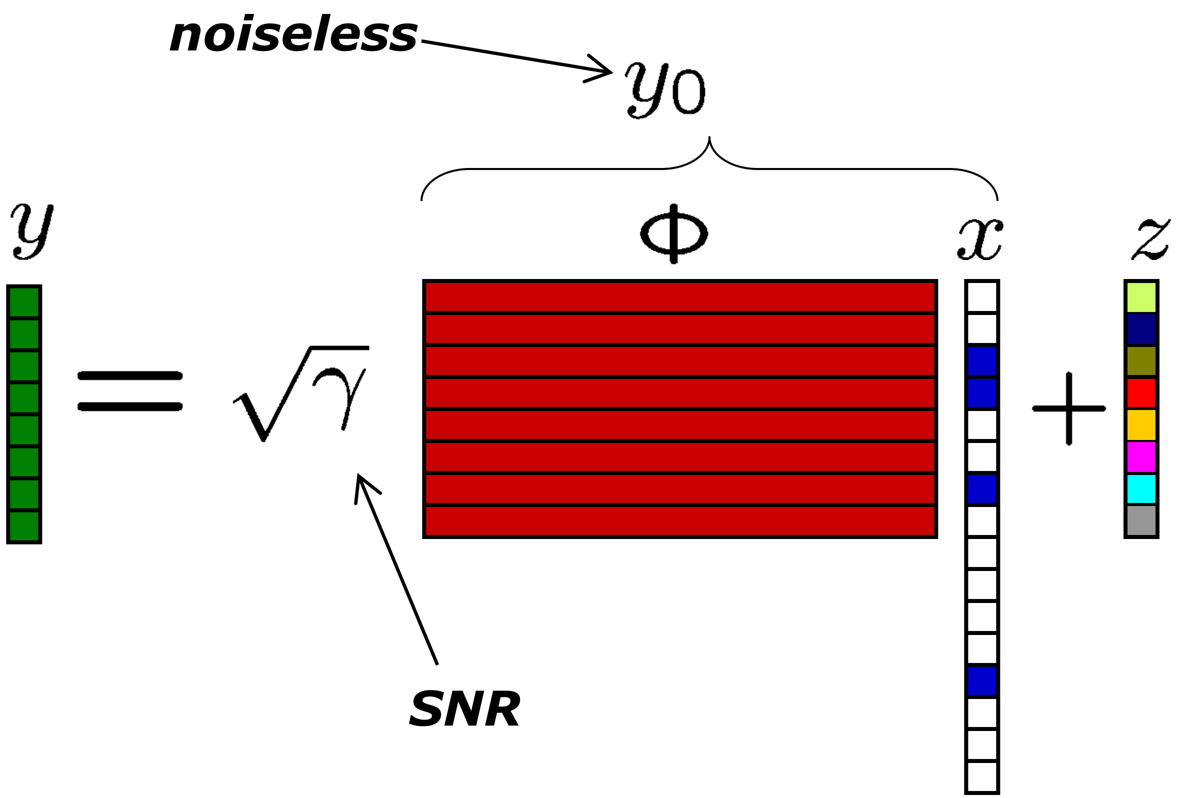
- *Assume Gaussian noise for ease of analysis*

- Can be generalized to non-Gaussian noise

  [Guo & Wang; Rangan]

Noise Model

- Noiseless measurements denoted y_0
- Noisy measurements $y = \sqrt{\gamma}\ \Phi x + z$
- Noise $z_m \sim \mathcal{N}(0,1)$
- Unit norm columns → γ is SNR

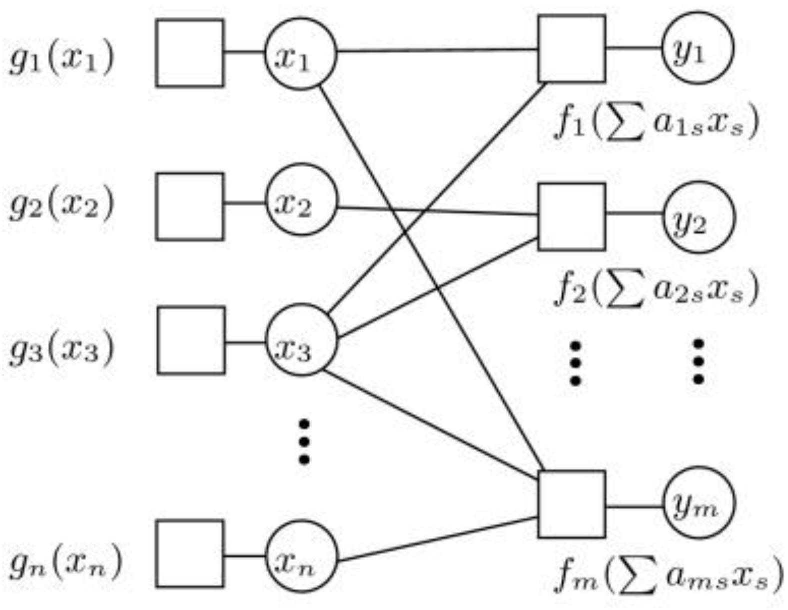


How do we Solve Fault Identification System?

[Bickson, B, Ihler,… 2011]

How Do We Identify the Faults?

- *Graphical model* relates fault pattern x to noisy measurements y using
- *How do we solve bipartite graph?*
- Direct evaluation of node
  - d incoming eges
  - ***combinatorial*** in d… ☹

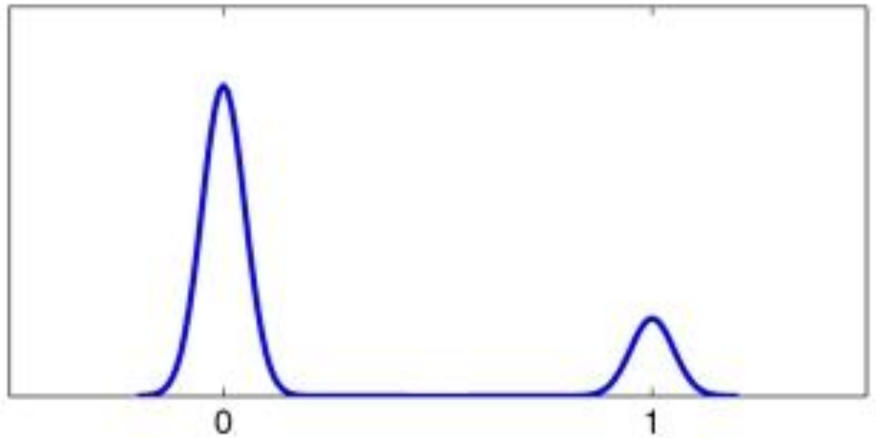


Solve Using Continuous Variables

- *Nonparametric belief propagation*
  - performs BP in continuous domain [Bickson et al., Sommer et al.,...]

- Using delta input pdf returns us to same problem

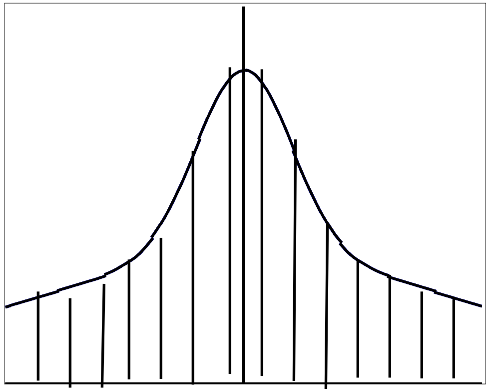
⇒ ***Relax*** *prior into two-state mixture Gaussian*



Representation of PDF's

- Messages (distributions) are continuous
- Need method to encode continuous messages

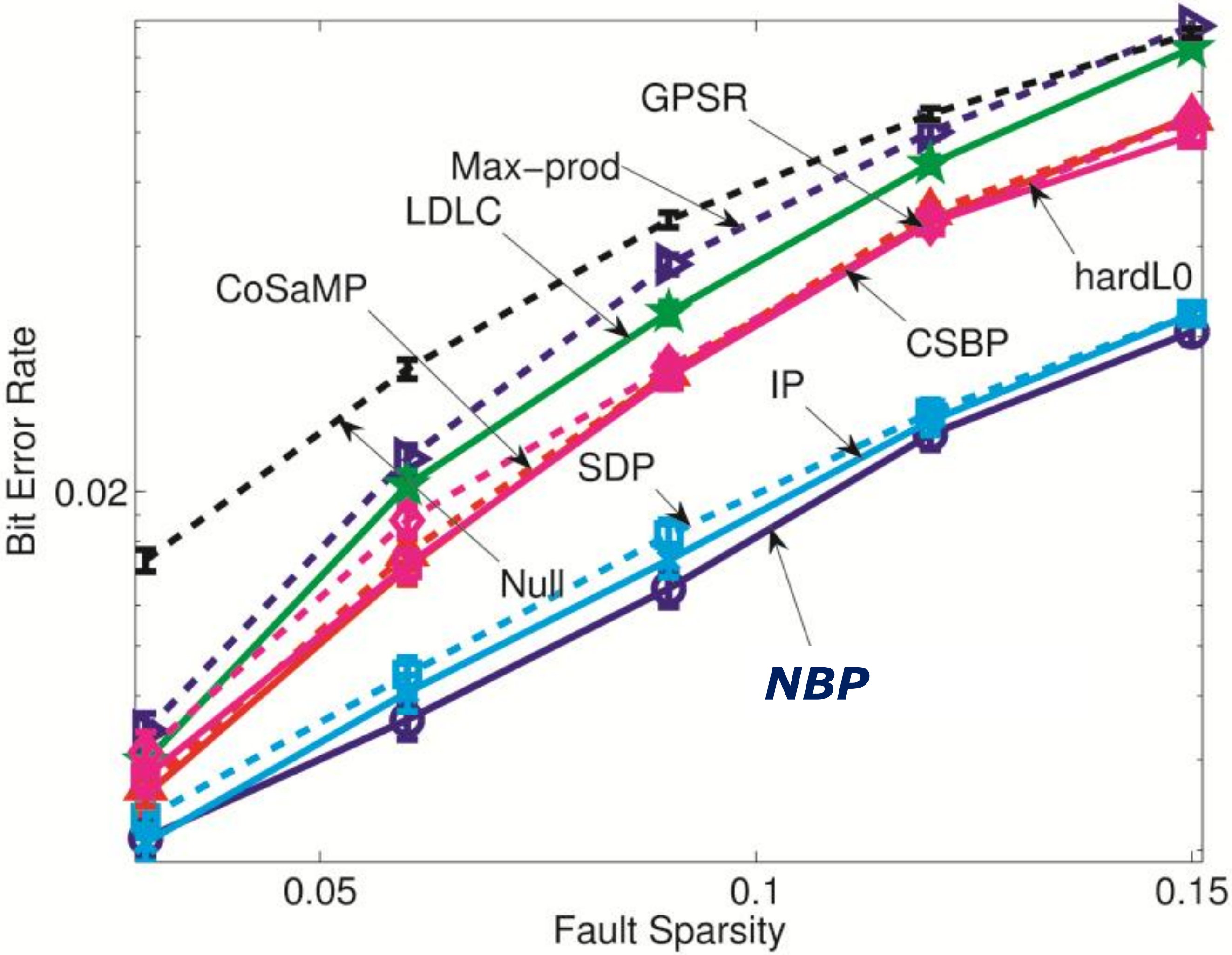
- *Uniform samples of PDF*
  [Sarvotham, B, Baraniuk 2006]
  [B, Sarvotham, Baraniuk 2010]
  - flexible to arbitrary distributions
  - requires lots of memory



- *Mixture model*
  - short messages (little memory)
  - inflexible; strong assumptions on distribution structure

Note: one mixture component asymptotically optimal in large system limit [Donoho, Maleki, & Montanari 2009; Rangan 2010]

NBP vs. Other Algorithms [N=100,M=50,q=0.2]



COMM algos only uses Bernoulli

CS algos only sparsity

optimization algorithms

- *NBP exploits **both** sparsity and Bernoulli*

Why Does it Work?

[Guo, B, & Shamai 2009]

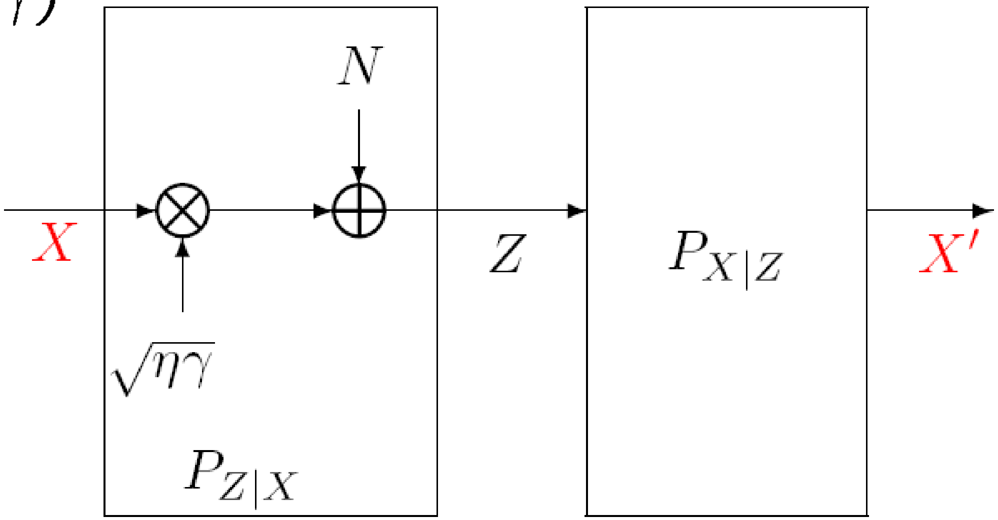
Information Theoretic Characterization

[Tanaka 2002] [Guo & Verdu 2005] [Guo & Wang 2008] [Guo, B, & Shamai 2009] [Donoho, Maleki, & Montanari 2009] [Bayati & Montanari 2009][Rangan, Fletcher, & Goyal 2009], [Rangan 2010]

- **Theorem:** Conditioned on $X_n=x_n$, observations (Y,Φ) statistically equivalent to $Z_n \sim \mathcal{N}(\sqrt{\eta\gamma}x_n, 1)$

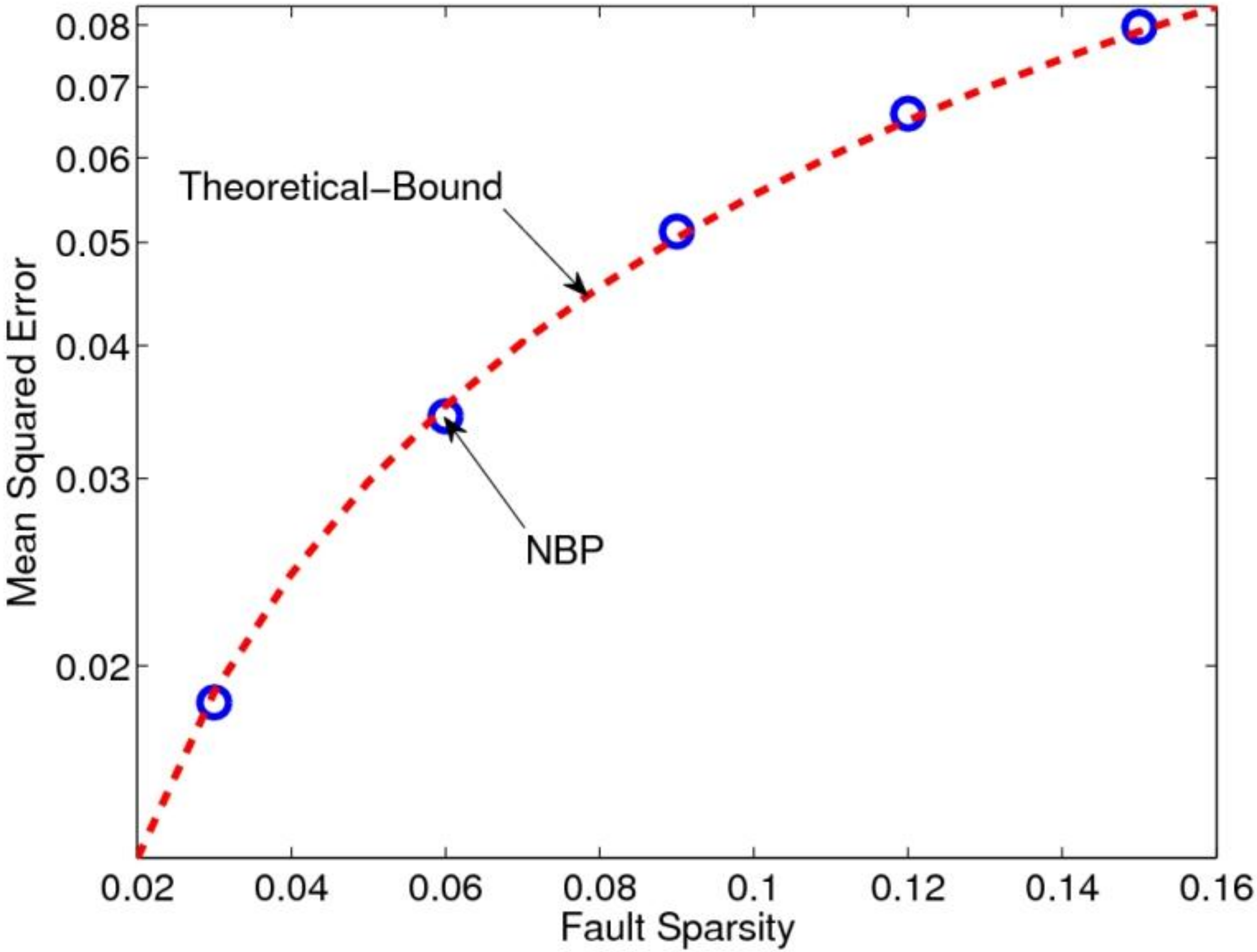
- SNR degradation η obeys fixed point equation

$$\eta^{-1} = 1 + \frac{\gamma}{\delta}\text{mmse}(P_X, \eta\gamma)$$



- Estimation quality as good as *degraded scalar channel*

NBP ***Delivers!*** [N=500,M=250,q=0.1]



Final Words

- Fault identification consists of superposition of (sparse) faults
- Relaxing Bernoulli structure into two-state mixture allows to use continuous NBP
- NBP exploits both sparsity and Bernoulli
- Achieves information theoretical limit

THE END