
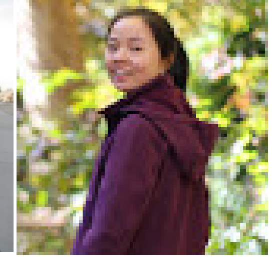

Signal Estimation with Mixtures

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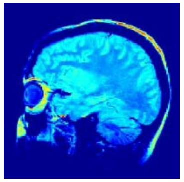
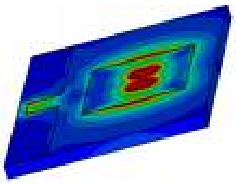
Signal Estimation

- Medical imaging (tomography)



- Source and channel coding

- Financial prediction
- Electromagnetic scattering

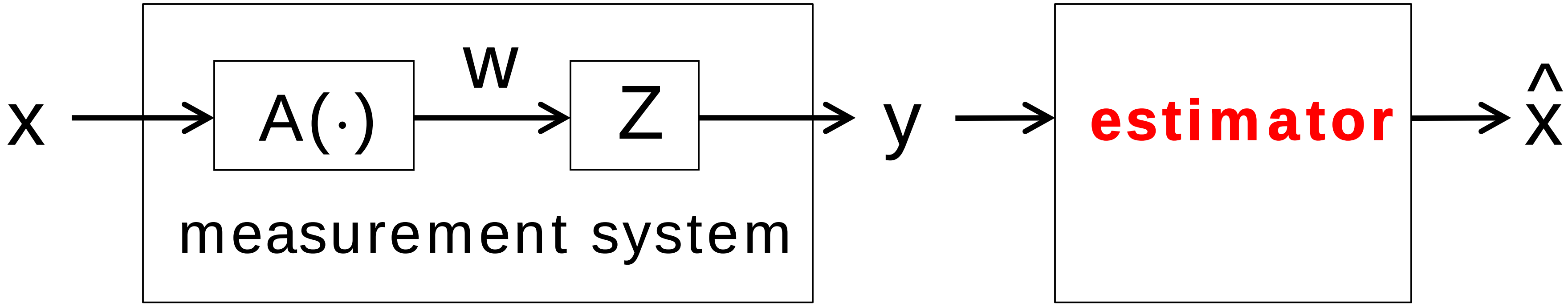


- Seismic imaging (oil industry)

- Speech recognition
- *Many more...*

Problem Setting

- Real-valued input x (vector); *unknown* distribution
- *Measurement system* A
  - identity operator (w=x)
  - matrix-vector product (w=A*x)
  - nonlinear (w=A(x))
- *Noise* (additive, $f_{Y_i|W_i}$, non-iid,...)
- *Measurements* y
- *Goal*: estimate x from y, A, f_Z



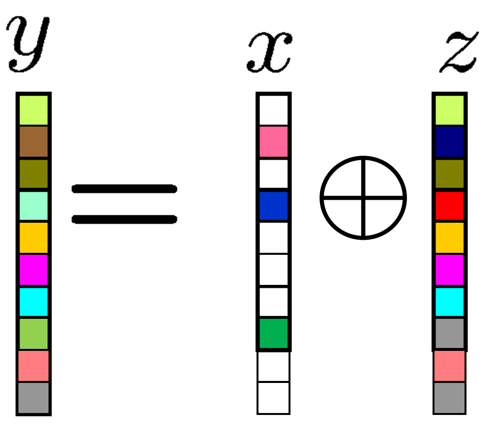
Scalar Channel

Scalar Channel

- Length-N *input* vector x
- Additive *noise* z
- *Observations* y=x+z

- **Goal**: estimate x from y

- Will use *mean square error* criteria

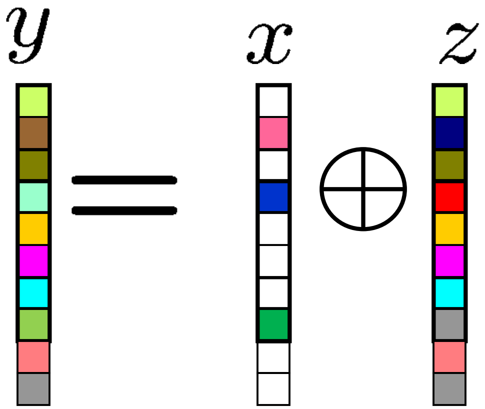


Bayesian Denoising [Bayes 1763]

- View X and Z as random vectors
- Known distributions
- $X \sim f_X(x)$, $Z \sim f_Z(z)$

- Conditional expectation achieves minimum mean square error (MMSE)

$$E[X|Y] = \int x \, \mathbf{f_X(x|y)} \, dx$$



Parametric Input Distribution

- Prior $f_X(x)$ often *unknown*
- To keep things simple...
  - suppose prior is parametric distribution $f(x|\theta)$

- *What's Ɵ???*

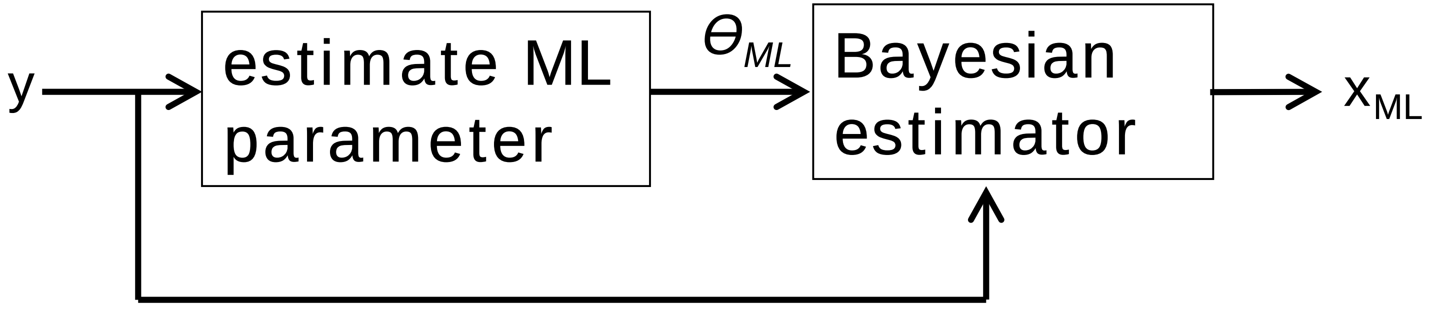
Plug-in Estimator

- Step 1: compute ML parameter

  $\Theta_{ML} = \text{argmax}_{\Theta} f(y|\Theta)$

- Step 2: plug Θ_{ML} into Bayesian estimator

  $x_{ML} = E[X|Y,\Theta_{ML}] = \int x\, \mathbf{f_X(x|y,\Theta_{ML})}\, dx$

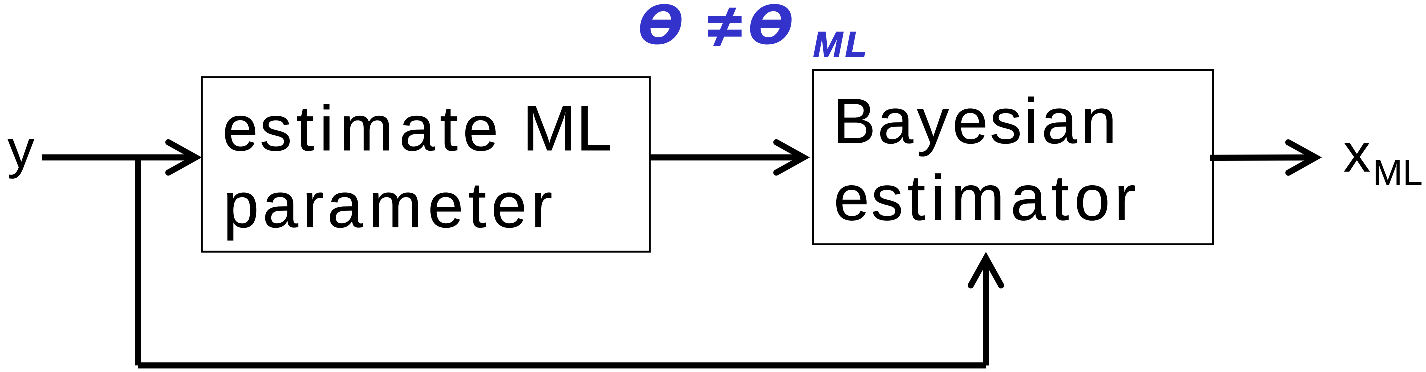


Limitation

- ML parameter (usually) converges to true Θ for large N

- For small N, *mismatch* between Θ and Θ_{ML} will increase mean square error (MSE) [Verdú 2010]

- Plug-in discards all other Θ that might also be informative



Achieving MMSE with Mixtures

Achieving MMSE with Mixtures

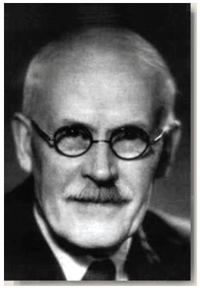
Proposed by Neri Merhav

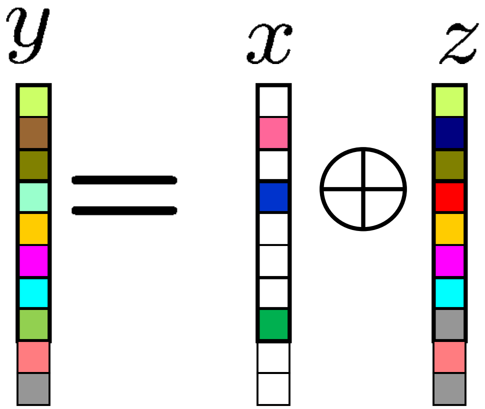
Mixture Universal Signal Estimator (MUSE)

- Scalar channel y=x+z
- Conditional expectation using mixture over Θ

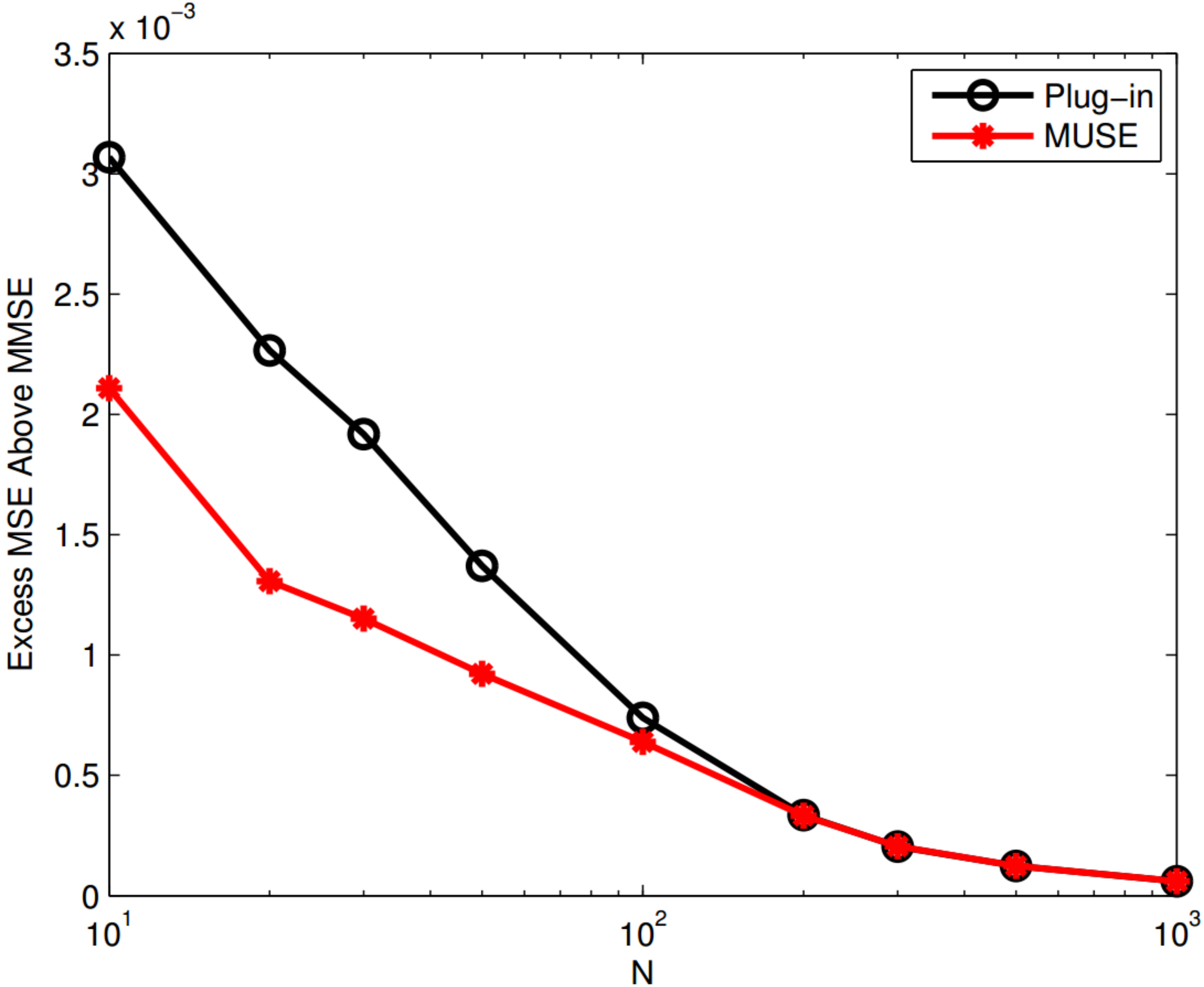
  $$E[X|Y] = \int E[X|Y,\Theta]\, \mathbf{f(\Theta|y)}\, d\Theta$$

- Posterior $f(\theta|y) = \frac{f(y|\theta)f(\theta)}{\int f(y|\theta')f(\theta')d\theta'}$
- Need objective prior for Θ, e.g., Jeffreys' prior





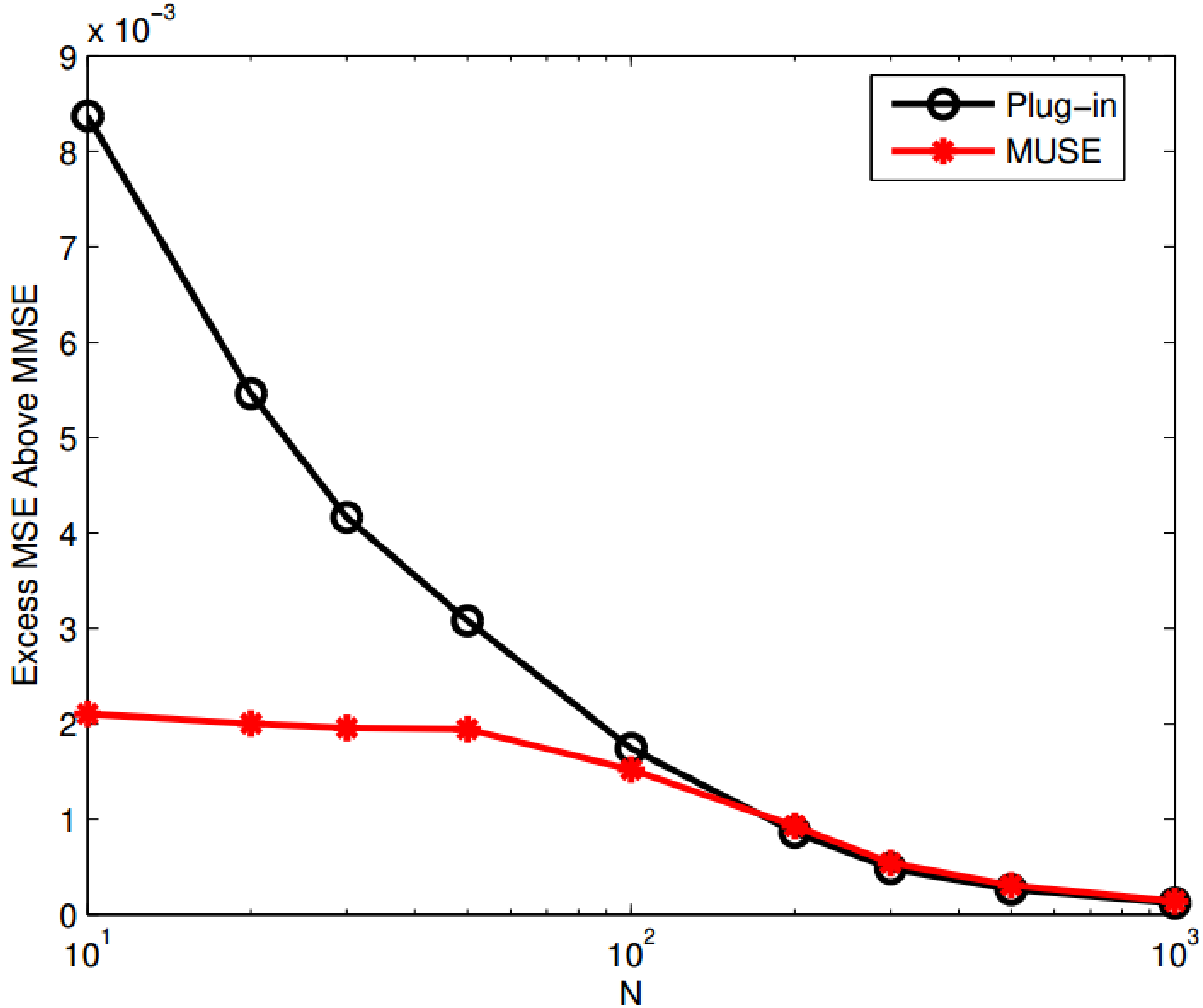
Bernoulli Input (ϴ=0.1, Var(Z)=0.1)



Bernoulli Gaussian Input

$$f(x|\theta,\mu,\sigma_x) = \theta \cdot N\left(\mu,\sigma_x^2\right) + (1-\theta)\cdot\delta(x)$$

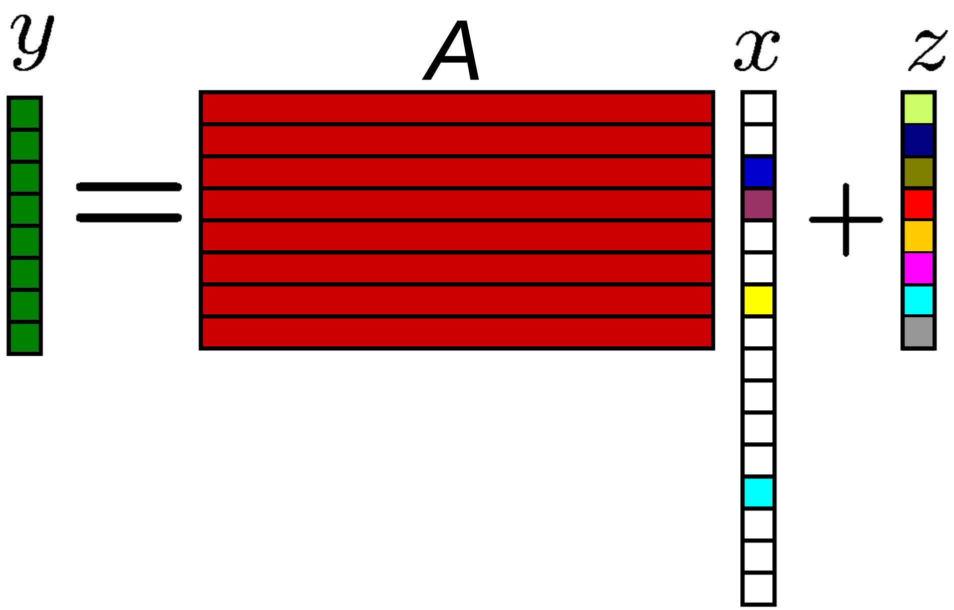
$$\theta = 0.1;\ \mu = 0; \mathrm{Var}(X) = 0.01, \mathrm{Var}(Z) = 0.1$$



Matrix Channel

Example: Compressed Sensing

- Length-N input, $x \sim f_X(x|\Theta)$ (typically sparse)
- Matrix A
- Additive noise, $z \sim f_Z(z)$ (e.g., i.i.d. Gaussian)
- Modest # measurements M suffices for robust signal reconstruction



Approximate Message Passing (AMP)

[Donoho, Maleki, & Montanari 2009]

AWGN channel

$$X^{t+1} = \eta_t(X^t + A^T r^t) = \underbrace{\eta_t(X + N(0, \sigma_t^2))}$$

$$r^t = y - Ax^t + \underbrace{\frac{r^{t-1}}{M/N} < \eta'_{t-1}(X^{t-1} + A^T r^{t-1}) >}$$

correction term

AMP-*MUSE*

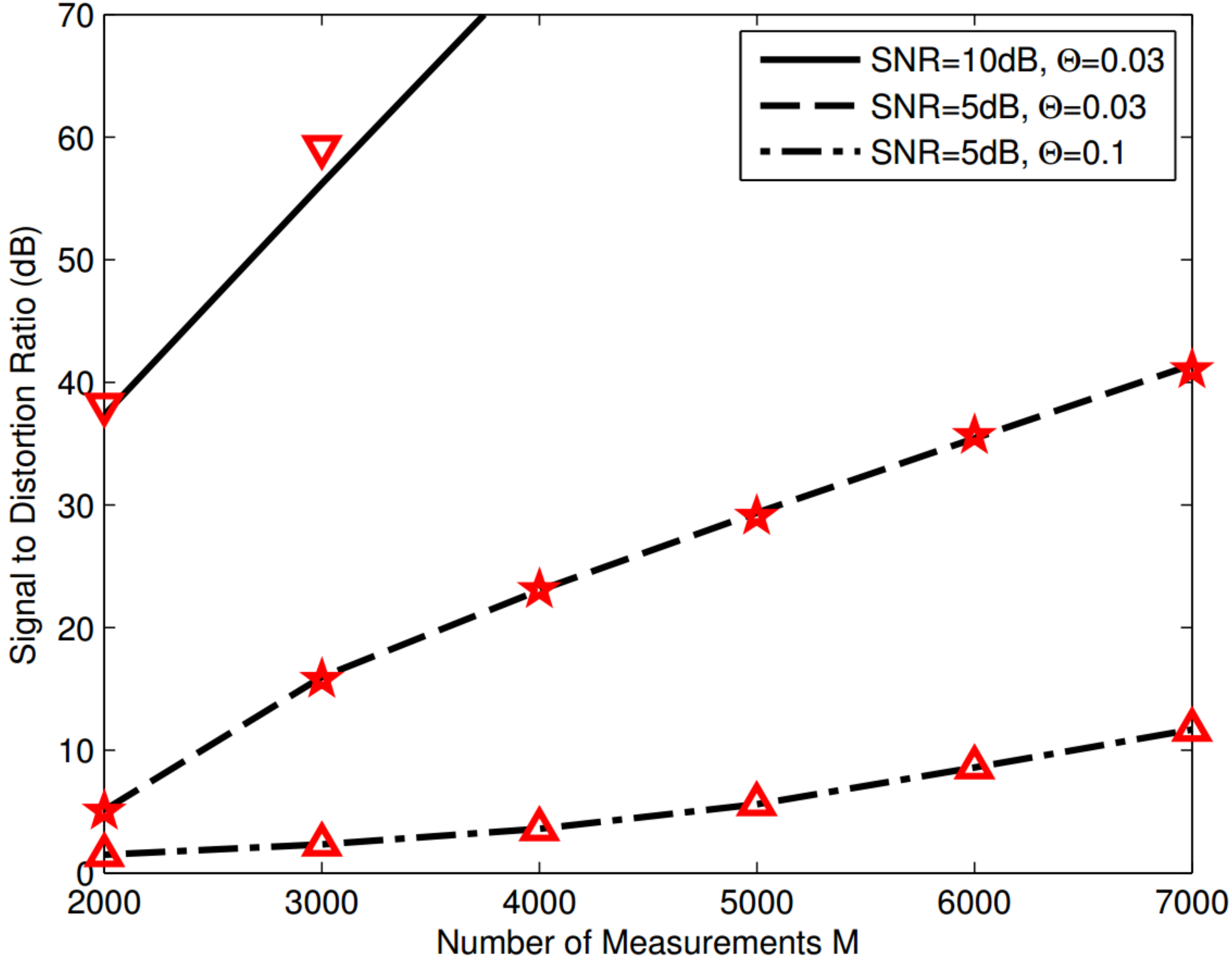
- Following [Donoho et al. 2013] use non-separable MUSE denoiser within AMP

MUSE denoiser

$$X^{t+1} = \eta_t(X^t + A^T r^t) = \eta_{\mathbf{MUSE},t}(X + N(0, \sigma_t^2))$$

$$r^t = y - Ax^t + \frac{r^{t-1}}{M/N} < \eta'_{\mathbf{MUSE},t-1}(X^{t-1} + A^T r^{t-1}) >$$

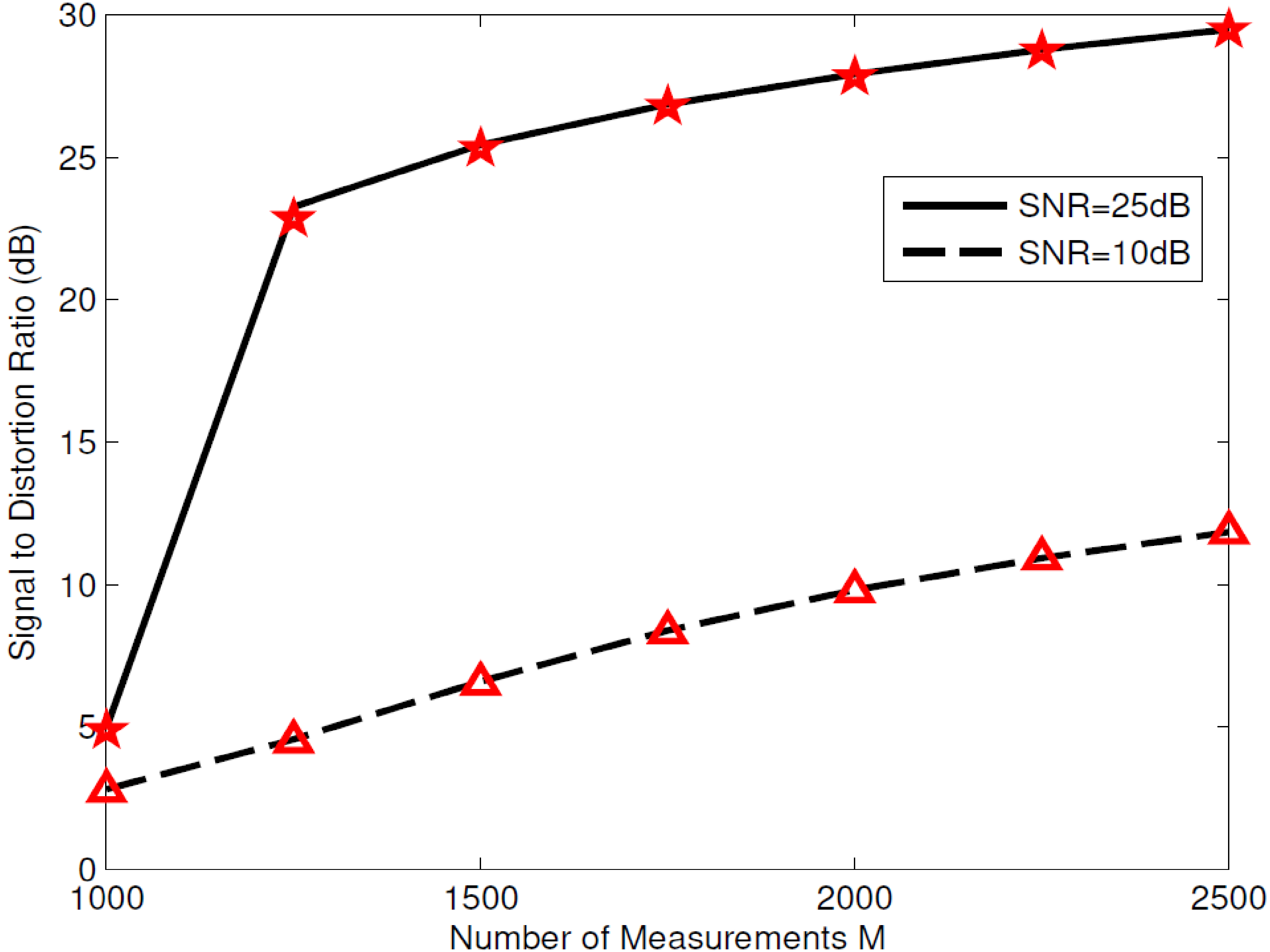
Bernoulli Input with AWGN (N=10,000; i.i.d. matrix)



Bernoulli Gaussian Input with AWGN

$$f(x|\theta,\mu,\sigma_x) = \theta \cdot N(\mu,\sigma_x^2) + (1-\theta) \cdot \delta(x)$$

$$N = 5{,}000;\ \theta = 0.1;\ \mu = 0;\ \sigma_x = 1$$



Final Words

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- Mixture universal signal estimation (MUSE)
  - demonstrated for two toy models
  - approaches MMSE
  - beats plug-in for small problem sizes

- Easily applied to matrix channels (AMP-MUSE)

- Open problem: *optimal* universal signal estimation
  - how quickly can we approach MMSE? (Rissanen-style bounds)
  - what algorithms achieve these bounds?

- *This is ongoing work – many open problems!*

THE END