Maximum Likelihood Quantum Error Mitigation for Algorithms with a Single Correct Output

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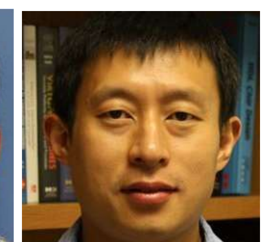
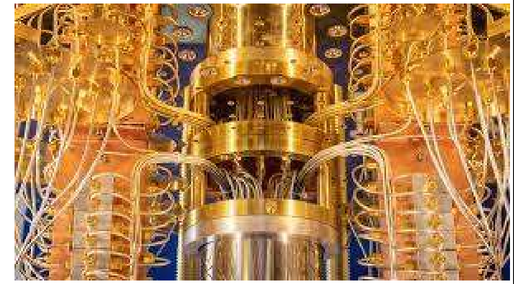


Motivation



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- Quantum computing can provide major speedups over classical (non-quantum) counterparts
- However…

Motivation

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- Quantum computing can provide major speedups over classical (non-quantum) counterparts
- However…
- Quantum computations are noisy
  - Classical transistors $\sim 10^{-27}$ error rate
  - Quantum computations $\sim 10^{-4}$

Motivation

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- Quantum computing can provide major speedups over classical (non-quantum) counterparts

- However…
- # qubits increasing
  - Currently hundreds
  - 10X per 2-3 years

Current paradigm

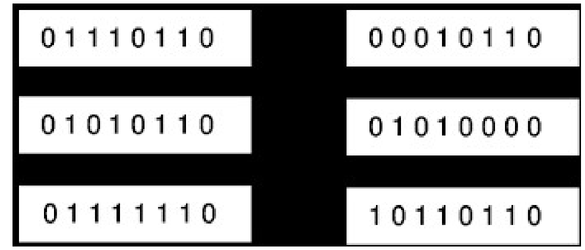
- Run S shots of quantum device
  - Shots are pricey (e.g., $0.03 on Amazon Braket)
- After each shot, measure n qubits
- Error mitigation extracts information from (many) noisy shots

Challenges

- Run S shots of quantum device
  - Shots are pricey (e.g., $0.03 on Amazon Braket)
- After each shot, measure n qubits
- Error mitigation extracts information from (many) noisy shots
- *How to extract information? How many shots?*

How to extract information from noisy shots?

Problem formulation



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- Quantum computer generates single correct bitstring $x_0 \in \{0,1\}^n$
  - Many quantum algorithm can generate many possible outputs
  - Bernstein-Vazirani has single output
  - Grover search may have single output


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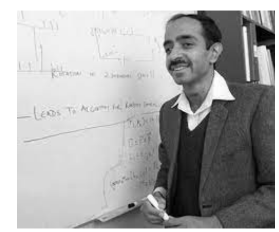
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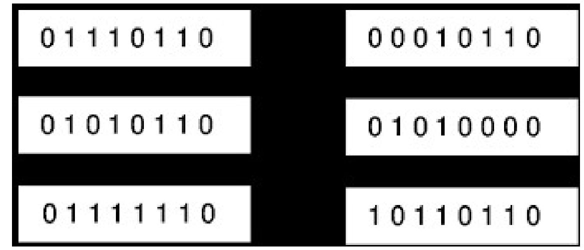
Problem formulation



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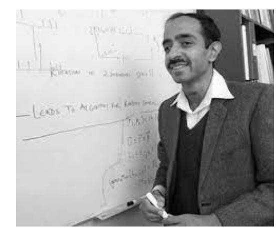
- S shots, $s \in \{1, \ldots, S\}$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- Noisy observation $y_s = x_0 \oplus noise_s$, $noise_s \in \{0,1\}^n$

What noise?

- Recall $y_s = x_0 \oplus noise_s$
- Symmetric i.i.d. noise: bit flips with probability p
  - We're *assuming* symmetric i.i.d.; more later
  - $Pr(y(i)=1|x_0(i)=0)=p$
  - $Pr(y(i)=0|x_0(i)=1)=p$
  - $p<0.5$

How to estimate x_0?

- With S shots, take mode (most common) length-n measurement
- For symmetric i.i.d. noise, mode identifies x_0 as $S \to \infty$

- Pro: simple estimator

- Con: S must be large

Can we do better than mode?

- Matrix free error mitigation (M3)
  [Nation et al. 2021]
- Considers transition probabilities between all observed strings
- Must observe x_0 at least once
- S exponential in n

Scalable mitigation of measurement errors on quantum computers

Paul D. Nation,* Hwajung Kang, Neereja Sundaresan, and Jay M. Gambetta
IBM Quantum, Yorktown Heights, NY 10598 USA
(Dated: August 31, 2021)

We present a method for mitigating measurement errors on quantum computing platforms that does not form the full assignment matrix, or its inverse, and works in a subspace defined by the noisy input bit-strings. This method accommodates both uncorrelated and correlated errors, and allows for computing accurate error bounds. Additionally, we detail a matrix-free preconditioned iterative solution method that converges in $\mathcal{O}(1)$ steps that is performant and uses orders of magnitude less memory than direct factorization. We demonstrate the validity of our method, and mitigate errors in a few seconds on numbers of qubits that would otherwise be intractable.

arXiv:2108.12518v1 [quant-ph] 27 Aug 2021

I. INTRODUCTION

Recently, rapid developments in the fabrication, control, and deployment of quantum computing systems has brought qubit counts to ~ 100, where it might be possible to show advantage over classical computation methods in one or more limited cases [1–3]. However, such breakthroughs are hampered by noise and errors that conspire to limit the effectiveness of quantum computers at tackling problems of appreciable scale. To counteract these effects, researchers have turned to mitigation methods that approximately cancel quantum gate [4–10] and measurement assignment [11–19] errors. For short-depth quantum circuits that can be executed on current generation hardware, measurement errors play an out-sized role, and their correction is critical to many near-term experiments [20–27].

In the canonical situation where initialization noise is minimal, measurement errors over N-qubits can be treated classically and satisfy

$$\vec{p}_{\mathrm{noisy}} = A\vec{p}_{\mathrm{ideal}}, \quad (1)$$

where $\vec{p}_{\mathrm{noisy}}$ is a vector of noisy probabilities returned by the quantum system, $\vec{p}_{\mathrm{ideal}}$ is the probabilities in absence of measurement errors (but still includes e.g. gate errors), and A is the $2^N \times 2^N$ complete assignment matrix (A-matrix) where element $A_{\mathtt{row},\mathtt{col}}$ is the probability of bit-string `col` being converted to bit-string `row` by the measurement error process [see App. (A) for examples]. While computing A requires executing 2^N circuits, it is often the case that errors on multiple qubits can be well approximated using at most $\mathcal{O}(N)$ calibration circuits; the A-matrix can be approximated efficiently.

Equation (1) has a solution $\vec{p}_{\mathrm{ideal}}$ readily found using

$\vec{p}_{\mathrm{noisy}}||_2^2$, where $\vec{p}_{\mathrm{ideal}}$ is constrained to be positive, is often used in place of a direct solution [12–14, 16, 28]. Although physically appealing, the run times of these methods are orders of magnitude longer than those of direct techniques. Alternatively, it has been shown that quasi-probabilities can be used provided that one mitigates expectation values [4, 11, 29]. As proven in Ref. [11], these quasi-probabilities provide an unbiased estimate for the expectation value ξ of an operator O, with a spectral radius of one, that is diagonal in the computational basis

$$\xi = \sum_{i=0}^{2^N-1} \left[OA^{-1}\vec{p}_{\mathrm{noisy}}\right]_i . \quad (2)$$

Near-term algorithms such as the ubiquitous Variational Quantum Eigensolver (VQE) [20, 30] and quantum machine learning [22, 27] rely on the computation of expectation values, making the correction of measurement errors in these quantities an important step along the road to quantum advantage.

Current measurement mitigation techniques utilize the full 2^N-dimensional probability space, and thus do not scale beyond a handful of qubits. A truncation scheme was developed in [23], however it did so at the loss of measurement information, and still required explicit construction of the full A-matrix. Creating a scalable mitigation strategy requires reducing the dimensionality of the linear system in Eq. (1) without the need for computing A itself. Fortunately, present day cloud-accessible quantum computing systems have measurement error rates of a few-percent or less, see Table I, indicating it is possible to view the measurement error process as a small correction to the ideal probability distribution; measurement

Do we ***really*** need to observe x_0?

Toy example

- n=5 qubits
- Correct bitstring x0 = 0 0 0 0 0
- Noisy observations (S=5 shots)
- y_1= 0 0 0 0 1
- y_2= 0 0 0 1 0
- y_3= 0 0 1 0 0
- y_4= 0 1 0 0 1
- y_5= 1 0 0 0 0
- Focusing on qubit #4, it was mostly measured correctly

Parrot example

- n=1000 qubits
- Io Parrot likes messy environment
  - She flips 5-10% of bits

- Each y_s has many errors
- Very unlikely to ever sample correct x_0

- Take S=100
- *Majority vote should work well*

Qubit-wise majority vote (QMV)

- For each qubit, $i \in \{1, \ldots, n\}$, look at S corresponding bits
- Vote majority among S bit observations

✓Simple

✓No need to observe x_0

✓QMV = maximum likelihood (ML) for symmetric i.i.d. noise

✓QMV is classical algorithm; more possible applications

How many shots does QMV need?

Error analysis arxiv.org/abs/2402.11830

- Error = at least S/2 bits flipped
- Showed Pr(S/2 flipped)=$(4p(1\text{-}p))^{S/2}$poly(S)
- What about >S/2 bits flipped?
  - Multiplicative factor $\frac{1-p}{1-2p}$
- S logarithmic in n makes Pr(error) small enough

Required S arxiv.org/abs/2402.11830

- Define p=0.5-ε; how close we are to 50-50
- Need S=$\frac{0.5\ln(n)}{\varepsilon^2}$

- Pro: logarithmic in n

- Con: p might be close to 0.5

Numerical results

Typical result arxiv.org/abs/2402.11830

- Ran on ibm_Sherbrooke quantum machine
- n=20 qubits
- Correct output x_0=01010101010101010101
- Mistakes in red

Shots S	Mode	M3	QMV (new)
1024	01110101010111110101	01110101010111110101	01010101010101010111
4096	01010111000101011101	01010111000101011101	01010101010101010111
16384	01010101011101010111	01000101010101010101	01010101010101010101

- QMV identified x_0 perfectly; qubit #19 corrected

Discussion

Discussion

- Majority vote > M3 > mode
- Quantum machines don’t have symmetric i.i.d. bit flips
  - Asymmetric? See paper (weighted vote)
  - Some noisy individual qubits? QMV sensitive to this scenario
- Multiple correct outputs? See paper + ongoing work

Thanks!