ECE 301 (Section 003) Homework 2
Fall 2026, Dr. Dror Baron
Graduate TA in Charge: Xiaowen Tian
Maximum: 160 + Bonus 10

Problem 1 (Complex Exponentials)(20 points)

a) What is the signal $y(t) = Ce^{at}u(t)$ as shown in Figure 1? That is, use your new understanding of complex exponential signals to determine A, θ, r, and ω such that $C = Ae^{j\theta}$ and $a = r + j\omega$. Note from the plots that $\mathrm{Re}\{y(0)\} = 0$ and $\mathrm{Im}\{y(0)\} = 2$.

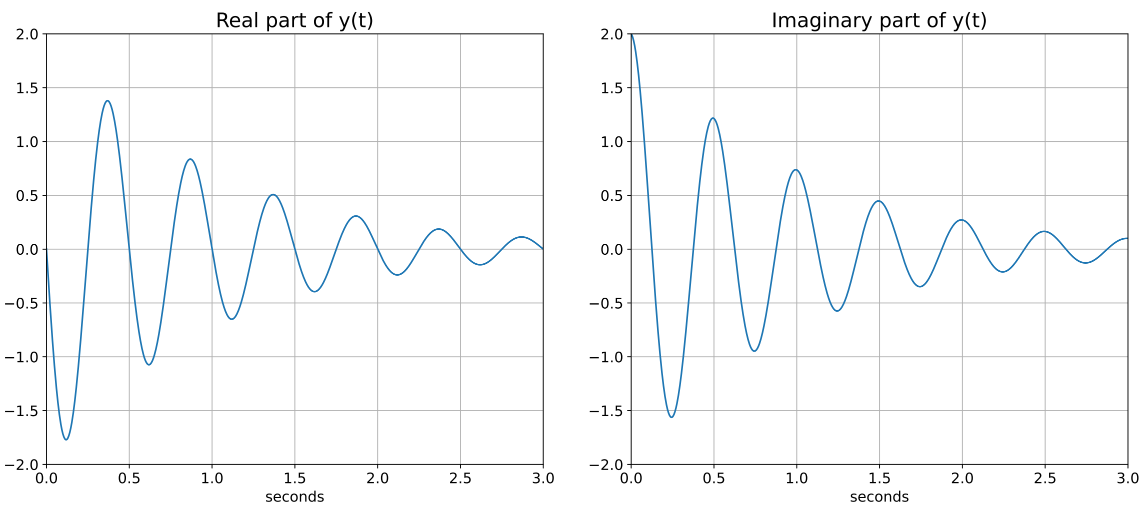


Figure 1: Continuous-time complex exponential, $y(t)$.

b) Determine the discrete time signal $y[n] = C\alpha^n u[n]$ shown in Figure 2. That is, determine A, θ, R, and ω_0 such that $C = Ae^{j\theta}$ and $\alpha = Re^{j\omega_0}$. You can assume $\mathrm{Re}\{y[0]\} = \mathrm{Im}\{y[0]\} = 1/\sqrt{2}$. (Hint: The trigonometric identity, $\cos^2\phi + \sin^2\phi = 1$, may be helpful.)

Problem 2 (The Periodicity of a Discrete-Time Signal)(20 points)

a) Determine whether or not the following signals are periodic. If periodic, determine its fundamental period, otherwise explain why it is aperiodic.

i) $x[n] = \cos(n/6 + \pi/4)$

ii) $x[n] = e^{j\frac{3\pi}{2}n} + e^{j\frac{5\pi}{3}n}$

b) Use Matlab to plot the real parts of the above signals to verify your analytic results. Append your code and plots in your submission. Add whitespace when necessary to enhance the readability of your code.

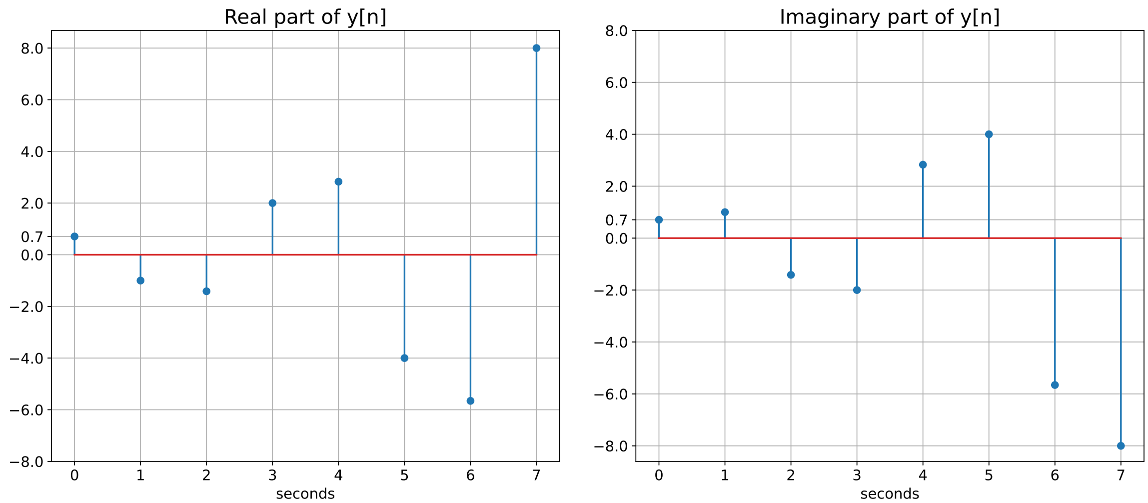


Figure 2: Discrete-time complex exponential, $y[n]$.

c) (Bonus, 5 pts) Consider the periodic discrete-time exponential time signal

$$x[n] = e^{jm(2\pi/N)\,n}.$$

Show that the fundamental period of this signal is

$$N_0 = N/\gcd(m, N),$$

where $\gcd(m, N)$ is the greatest common divisor of m and N—that is, the largest integer that divides both m and N an integral number of times. For example,

$$\gcd(2, 3) = 1, \quad \gcd(2, 4) = 2, \quad \gcd(8, 12) = 4.$$

Note that $N_0 = N$ if m and N have no factors in common.

Problem 3 (Basic Operations on Signals)(20 points)

a) A discrete-time signal $x[n]$ is shown in Figure 3(a). Sketch carefully each of the following signals by hand:

i) $x[2n]$

ii) $x\left[\frac{n}{2}\right]$

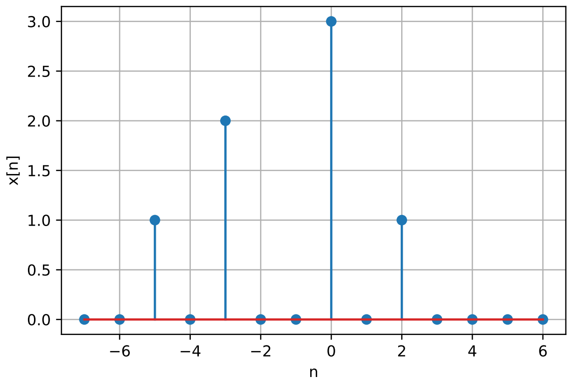
iii) $x[n]\,(u[n] + \delta[n-2])$

b) A continuous signal $x(t)$ is shown in Figure 3(b). Sketch carefully each of the following signals by hand:

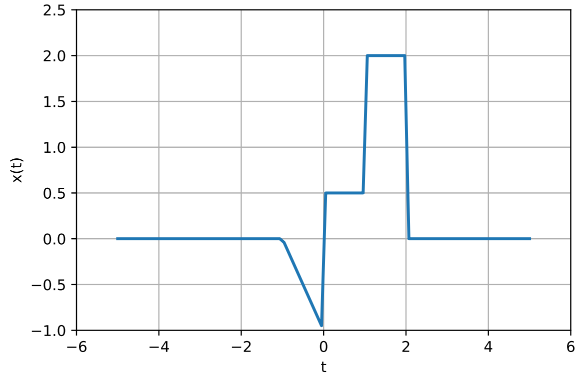
i) $x(t-3)$

ii) $x(1-t/2)$

iii) $x(t)\,[u(-t) + \delta(t-4)]$



(a)



(b)

Figure 3: (a) Discrete-time signal, $x[n]$. (b) Continuous-time signal, $x(t)$.

Problem 4 (Sifting Property of the Impulse Functions)(20 points)

a) Simplify the following expressions:

i) $\left(\frac{\sin(n+\pi)+8}{n^3-2}\right)\delta[n]$

ii) $\left(\frac{\cos(n\pi+\pi)}{-(n+1)^2}\right)\delta[n-3]$

iii) $\left(\frac{\cos t}{t^{64}-4}\right)\delta(t)$

iv) $\left(\frac{\sin(2k\omega)}{\omega}\right)\delta(\omega)$

b) Evaluate the following integrals and sums:

i) $\sum_{n=-\infty}^{\infty} \delta[n-6]\gamma^{n-3}\cos\left(\frac{\pi}{12}(6+n)\right)$

ii) $\int_{-\infty}^{\infty} \delta(t-4)\sin(\pi t)dt$

iii) $\int_{-\infty}^{\infty} \delta(3-t)f(1+t^2)dt$

iv) $\int_{-\infty}^{\infty} \delta(x-2)e^{(x-1)}\cos\left(\frac{\pi}{2}(x^2-5x+4)\right)dx$

Problem 5 (Impulse Function and Step Function) (20 pts)

a) Find and sketch the first derivatives of the following signals:

i) $x(t) = u(t) - u(t-a)$, $a > 0$,

ii) $x(t) = t[u(t) - u(t-a)]$, $a > 0$.

iii) $x(t) = \text{sign}(t) = \begin{cases} 1, & t \geq 0, \\ -1, & t < 0. \end{cases}$

b) Given that the $\delta(\cdot)$ is the unit impulse function, prove the following property:

$$\int_{-\infty}^{\infty} \delta(ax)dx = \int_{-\infty}^{\infty} \frac{\delta(t)}{|a|}dt = \frac{1}{|a|},$$

where a is a scalar. Can you claim that $\delta(ax) = \frac{1}{|a|}\delta(t)$? If yes, in what sense? If no, why.

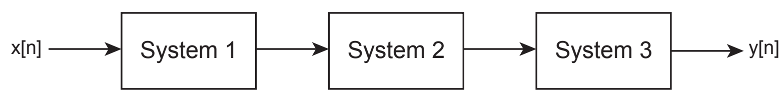
Problem 6 (Interconnected Systems) (20 pts) Consider three systems with these input–output relationships:

$$\text{System 1 (upsampling):} \quad w[n] = \begin{cases} x[n/3], & n \text{ multiple of } 3, \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{System 2 (smoothing):} \quad z[n] = w[n] + 2w[n-1] + w[n-2].$$

$$\text{System 3 (downsampling):} \quad y[n] = z[3n].$$

Suppose that these systems are connected in series, as shown below.



i) Find the input–output relationship, namely, write $y[n]$ as a function of $x[n]$, for the overall interconnected system.

ii) Is it time invariant? Is this system linear? (This subquestion should be answered without using the derivation steps. In other words, it should not be answered using the steps in Problems 7 and 8.)

(Hint: You can draw plots to see "physically," how a three-sample input signal, e.g., $x[0] = 1$, $x[1] = -1$, $x[2] = 2$, $x[n] = 0$ for all other n, evolves as it passes each block of the system.)

Problem 7 (Properties of Discrete Systems) (20 pts) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y[n]$ denotes the system output and $x[n]$ denotes the system input.

a) $y[n] = x[n]\,x[n-1]\,x[n+1]$

b) $y[n] = \begin{cases} x[n-1], & n \leq 0, \\ x[n+1], & n > 0. \end{cases}$

c) $y[n] = \cos(x[n])$

Problem 8 (Properties of Continuous Systems) (20 pts) Determine (with justification) whether the following systems are (i) memoryless, (ii) causal, (iii) invertible, and (iv) stable. For invertibility, either find an inverse system or an example of two inputs that lead to the same output. Note that $y(t)$ denotes the system output and $x(t)$ is the system input.

a) $y(t) = (t-2)\,x(t+2)$

b) $y(t) = x(t-1) + x(4-t)$

c) $y(t) = x\big(\sin(t)\big)$

d) **(Bonus, 5 pts)** $y(t) = \begin{cases} 0, & x(t) < 1, \\ \int_0^1 x(t-\tau)d\tau, & x(t) \geq 1. \end{cases}$