

ECE 301 (Section 003) Homework 6
Fall 2026, Dr. Dror Baron
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Maximum: 120 + Bonus 20

Problem 1 (20 pts) (Eigen-signals/functions of an LTI System) In the lecture, it was stated without proof that $e^{j\omega_0 t}$ is an eigen-signal of an LTI system $h(t)$. In other words, when the LTI system operates on an input signal $x(t) = e^{j\omega_0 t}$ [or $x(t)$ is sent into the LTI system], the output $y(t)$ is merely a scaled version of $x(t)$ for all $t \in \mathbb{R}$. Show that the scaling factor is

$$H(j\omega_0) = \int_{-\infty}^{\infty} h(t)e^{-j\omega_0 t} dt. \quad (1)$$

Recall that the input-output relation of an LTI system is related by $y(t) = h(t) * x(t)$.

Problem 2 (20 pts) (Fourier Transform) Compute Fourier transforms for the following signals. **Notice: you must do the complete integral computation without relying on the tables.**

- a) $x(t) = e^{-2(t-1)}u(t-1)$
- b) $x(t) = e^{-|t+2|/3}$ (Explicitly show how the absolute sign is removed.)
- c) $x(t) = \text{rect}(2t+1)$ (Final result must be represented in form of a sinc function.)
- d) $x(t) = 1+t$ for $t \in [-1, 0]$ otherwise $x(t) = 0$ (Show the details of integration by parts.)

Problem 3 (20 pts) (Inverse Fourier Transform and Properties) Compute the inverse Fourier transform for the following signals. You must calculate the results using both i) the direct evaluation method based on the definition of the inverse FT and ii) the table of Fourier transform properties.

- a) $\delta(\omega+1) + \delta(\omega-1) + j\delta(\omega+3) - j\delta(\omega-3)$
- b) $\text{rect}(3\omega-2)$

Problem 4 (Bonus, 20 pts) (Various Fourier Transform Properties)

- a) Use a procedure similar to what's covered in the lecture, prove that differentiation in frequency domain corresponds to multiplication by jt in the time domain, namely,

$$-jtx(t) \xleftrightarrow{\mathcal{FT}} \frac{dX(j\omega)}{d\omega}.$$

b) Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure 1. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform. [Hint: The subplot (d) may need the result proved in part a).]

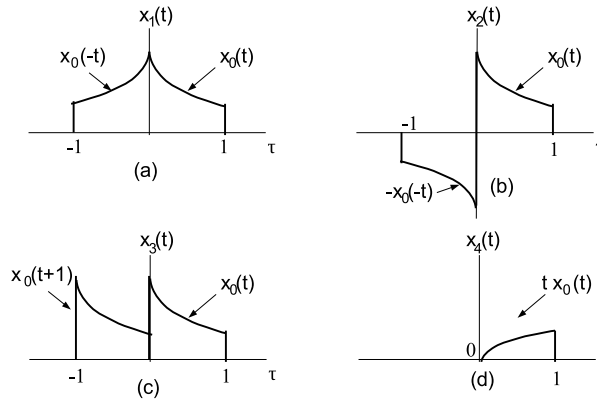


Figure 1: Signals for Fourier transform calculation.

Problem 5 (20 pts) (Windowed Time-Domain Signals) Find and plot the Fourier transform of the following windowed sinusoidal signals.

a)

$$x(t) = \begin{cases} \cos(10t), & -10 \leq t \leq 10. \\ 0, & \text{elsewhere.} \end{cases}$$

b)

$$x(t) = \begin{cases} \cos(10t), & 0 \leq t \leq 10. \\ 0, & \text{elsewhere.} \end{cases}$$

Hint: Use the multiplicative property of the Fourier transform.

Problem 6 (20 pts) (Lowpass Filter) A lowpass filter $H(\omega)$ has the frequency response shown in Figure 2.

a) Compute the impulse response $h(t)$ of the filter.

b) Compute the response $y(t)$ when the input is $x(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$, $-\infty < t < \infty$.

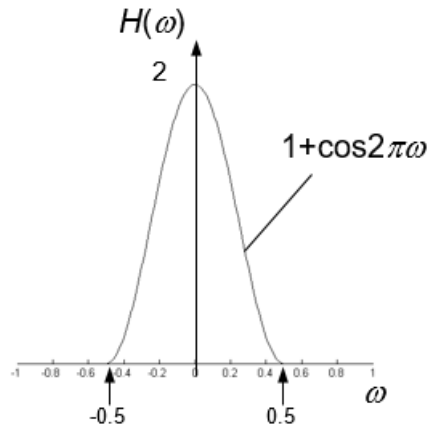


Figure 2: Frequency response of the lowpass filter $H(\omega)$.

- c) Compute the response $y(t)$ when $x(t) = \text{sinc}\left(\frac{t}{4\pi}\right)$, $-\infty < t < \infty$.

Note: You will need to use Fourier tables.

Problem 7 (20 pts) (DTFT and Inverse DTFT) Solve the following using the definitions **or** the properties.

- a) $\mathcal{F}\{u[n-2] - u[n-6]\}$
- b) $\mathcal{F}\{2^n \sin(\pi n/4) u[-n]\}$
- c) $\mathcal{F}\{\sin(\pi n/2) + \cos(7\pi n/3)\}$
- d) $\mathcal{F}^{-1}\left\{\frac{e^{-j\omega} - \frac{1}{3}}{1 - \frac{1}{3}e^{-j\omega}}\right\}$