

ECE 301 (Section 003) Homework 7
 Fall 2026, Dr. Dror Baron
 Graduate TA in Charge: Xiaowen Tian
 Maximum: 80 + Bonus 5

Problem 1 (20 pts) (Triangle, Rectangle, and Sinc Functions)

- a) Using the definition of convolution, show via integration that $y(t) = H \text{rect}(t/2W) * H \text{rect}(t/2W)$ is a triangle. What are the width and height of the triangle?
- b) Use the definition or the properties to compute the inverse DTFT of the following frequency functions $X(e^{j\Omega})$.

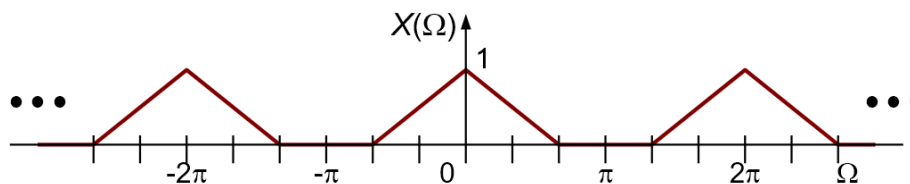


Figure 1: Frequency domain representation $X(e^{j\Omega})$ of a discrete-time signal $x[n]$.

Problem 2 (20 pts) (Windowing Effect and Frequency Resolution) In this problem, we will investigate the frequency resolution of Fourier transform. We investigate two neighboring musical notes, C_4 at $f_1 = 261.63$ Hz and $C_4^\#$ at $f_2 = 277.18$ Hz. You can play with a virtual piano here:

<https://recursivearts.com/virtual-piano/>

We make an oversimplified assumption that the audio signal of each musical note is in the form of $x_i(t) = \cos(2\pi f_i t)$, $t \in \mathbb{R}$. A causal time window $t \in [0, 2T]$ of unit gain is applied to “generate” signals of finite length. (To simplify the problem, we investigate the continuous-time signal. The analysis of the discrete-time signal is similar.)

- a) With the help of the CTFT table, calculate the Fourier spectra for the windowed $x_i(t)$, for $i = 1, 2$.
- b) What is the angular frequency of the first zero-crossing on the right-hand side of the magnitude spectrum of note C_4 ? What is the angular frequency of the first zero-crossing on the left-hand side of the magnitude spectrum of note $C_4^\#$?
- c) What is the smallest T that can clearly separate the peaks of the two neighboring musical notes?

Problem 3 (20 pts) (DFT Calculations and Convolution Property)

- a) Compute the DFT of the following signals by hand and plot $X[k]$. Label the low and high frequencies. Feel free to verify your results using Matlab command `fft`. For the following problems, we use vector notation to represent finite length signals. For example $\mathbf{x} = [-1, 2, 3]$ means a signal with $x[0] = -1$, $x[1] = 2$, and $x[2] = 3$.
- (i) DFT of $[1, 1, -1, -1]$.
 - (ii) DFT of $[1, 0, 1, 0]$.
 - (iii) Inverse DFT of $[0, 0, 1, 0]$.
- b) Show that the linear convolution result of $\mathbf{x} * \mathbf{y}$, where $\mathbf{x} = [1, 1]$ and $\mathbf{y} = [1, 1, -1, -1]$ is the same as the result of $\text{IDFT} \{ \text{DFT}([\mathbf{x}, 0, \dots, 0]) \text{DFT}([\mathbf{y}, 0, \dots, 0]) \}$. Note that you need to determine the correct numbers of zeros padded to $\mathbf{x}$ and $\mathbf{y}$, respectively. You may use Matlab command `fft` to help you calculate DFT.

Problem 4 (Bonus, 5 pts) (Computational Complexity in Time Domain vs. in Frequency Domain) In class, you've seen that calculating the convolution between two time-domain sinc functions in the frequency domain can significantly lower the computational cost. Argue under what scenario time-domain calculation can significantly lower the computational cost. Give a concrete example in terms of two signals.

Problem 5 (20 pts) (Laplace Transform) Determine the Laplace transform and the associated region of convergence (ROC) for each of the following functions of time. Provide the pole-zero plot including the ROC.

- a) $x(t) = e^{-3t}u(t) + e^{-4t}u(t)$.
- b) $x(t) = e^{-2t}u(t) + e^{-3t} \cos(2t)u(t)$.
- c) $x(t) = \delta(t) + u(2t)$.
- d) $x(t) = e^{-3|t|+1}$.