

ECE 301 – Linear Systems

Test 1 – Fall 2026

October 7, 2026

Please, remember to justify your answers carefully (except in Question 1). The test is 75 minutes. One cheat sheet is allowed.

Last name: _____ First name: _____

Question 1 (Short answer.)

No need to show work. Fill in the circle of your selected choice.

(a) A time-invariant system must satisfy the superposition property.

☐ True ☐ False

(b) The system $y(t) = x^3(t)$, where $x(t)$ is real-valued, is

☐ Invertible ☐ Not invertible ☐ Can't tell

(c) Convolution of a signal of length 3 samples with an impulse response of length 5 samples gives a signal of length

☐ 5 ☐ 7 ☐ 8 ☐ 9

(d) A causal system uses past, present, and future input values to determine the present output.

☐ True ☐ False ☐ Can't tell

(e) Let $x[n] = 2\delta[n+1] + 3\delta[n]$ and $h[n] = 4\delta[n-1] + 5\delta[n-2]$. The value of $y[1]$, where $y[n] = x[n] * h[n]$, is

☐ 8 ☐ 12 ☐ 22 ☐ 27

Question 2 (System properties.)

Consider the continuous-time system with input $x(t)$ and output $y(t)$, where $x(t)$ is real-valued,

$$y(t) = |x(t + 1)|.$$

Determine whether the system is **(a)** invertible, **(b)** time invariant, and **(c)** linear. Make sure to justify each answer with mathematical expressions and/or words; a counterexample is a fine justification where appropriate.

Question 3 (Periodicity and sifting.)

Throughout this question, t denotes continuous time and n denotes discrete time.

(a) Determine whether or not the following signals are periodic. If periodic, determine the fundamental period; otherwise, explain why the signal is aperiodic.

(i) $x(t) = \sin\left(\frac{4t}{9} - \frac{\pi}{6}\right)$. (ii) $x[n] = \cos\left(\frac{14\pi n}{15}\right)$.

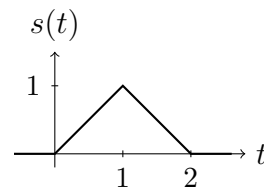
(b) Simplify/evaluate the following expressions.

(i) $\sum_{n=-\infty}^{\infty} (n^2 + 3n + 5) \delta[n + 1]$. (ii) $\int_{-\infty}^{\infty} \delta(t - 2) e^{t-2} \cos\left(\frac{\pi}{6}(t^2 - t)\right) dt$.

Question 4 (Step response and convolution.)

An LTI system has the triangular step response

$$s(t) = \begin{cases} t, & 0 \leq t < 1, \\ 2 - t, & 1 \leq t < 2, \\ 0, & \text{otherwise,} \end{cases}$$



i.e., the output when the input is $u(t)$.

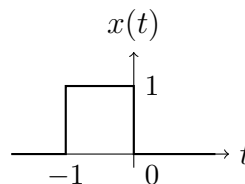
(a) Determine the impulse response $h(t)$, and sketch it.

Note: If you are unsure of your answer to part (a), you may instead use $h(t) = u(t) - u(t-2)$ in parts (b) and (d). In that case, your answer to part (d) will not agree with your answer to part (c).

(b) Using $h(t)$, determine whether the system is memoryless and causal.

(c) Let the input be

$$x(t) = u(t+1) - u(t) = \begin{cases} 1, & -1 \leq t < 0, \\ 0, & \text{otherwise,} \end{cases}$$



Find $y(t)$ using only $s(t)$ together with linearity and time invariance, and sketch $y(t)$.

(d) Now compute $y(t)$ by graphical convolution, flipping and sliding $x(t)$, i.e.,

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau.$$

Determine the ranges of t for which the formula for $y(t)$ is the same. For each range, set up the integral with explicit limits and evaluate it. Label at least one value on each axis of your plots. You have the rest of this page (page 5) and all of page 6 for your work on this part.

Question 4(d), continued.