

ECE 301 – Linear Systems

Test 1 – Fall 2026

October 7, 2026 – Solutions

Question 1 (Short answer.)

No need to show work. Fill in the circle of your selected choice.

(a) A time-invariant system must satisfy the superposition property.

☐ True ☐ False

Solution. False. Superposition relates to linearity, not time invariance; for example, $y(t) = x^2(t)$ is time invariant but not linear.

(b) The system $y(t) = x^3(t)$, where $x(t)$ is real-valued, is

☐ Invertible ☐ Not invertible ☐ Can't tell

Solution. Invertible. For real values, the cube is one-to-one, so $x(t) = \sqrt[3]{y(t)}$.

(c) Convolution of a signal of length 3 samples with an impulse response of length 5 samples gives a signal of length

☐ 5 ☐ 7 ☐ 8 ☐ 9

Solution. 7 samples, since the length is $L_x + L_h - 1 = 3 + 5 - 1 = 7$.

(d) A causal system uses past, present, and future input values to determine the present output.

☐ True ☐ False ☐ Can't tell

Solution. False. A causal system uses only present and past input values.

(e) Let $x[n] = 2\delta[n+1] + 3\delta[n]$ and $h[n] = 4\delta[n-1] + 5\delta[n-2]$. The value of $y[1]$, where $y[n] = x[n] * h[n]$, is

☐ 8 ☐ 12 ☐ 22 ☐ 27

Solution. 22. We have $y[1] = \sum_k x[k]h[1-k] = x[-1]h[2] + x[0]h[1] = 2 \cdot 5 + 3 \cdot 4 = 22$. (For reference, $y[n] = 8\delta[n] + 22\delta[n-1] + 15\delta[n-2]$.)

Question 2 (System properties.)

Consider the continuous-time system with input $x(t)$ and output $y(t)$, where $x(t)$ is real-valued,

$$y(t) = |x(t+1)|.$$

Determine whether the system is **(a)** invertible, **(b)** time invariant, and **(c)** linear. Make sure to justify each answer with mathematical expressions and/or words; a counterexample is a fine justification where appropriate.

Solution.

(a) Not invertible: $x_1(t) = 1$ and $x_2(t) = -1$ both give $y(t) = 1$, so the input cannot be recovered from the output.

(b) Time invariant. 1. Delay input: $x_2(t) = x(t - t_0)$. 2. Delay output: $y(t - t_0) = |x(t - t_0 + 1)|$. 3. Send delayed input to the system: $y_2(t) = |x_2(t + 1)| = |x(t + 1 - t_0)|$. 4. Check 2 & 3: they match, so the system is time invariant.

(c) Not linear. 1. Scale input: $x_2(t) = Ax(t)$. 2. Scale output: $Ay(t) = A|x(t + 1)|$. 3. Send scaled input to the system: $y_2(t) = |Ax(t + 1)| = |A| |x(t + 1)|$. 4. Check 2 & 3: for $A = -1$ and $x(t) = 1$, step 2 gives -1 while step 3 gives 1 . Scaling fails, so the system is nonlinear.

Question 3 (Periodicity and sifting.)

Throughout this question, t denotes continuous time and n denotes discrete time.

(a) Determine whether or not the following signals are periodic. If periodic, determine the fundamental period; otherwise, explain why the signal is aperiodic.

$$(i) \ x(t) = \sin\left(\frac{4t}{9} - \frac{\pi}{6}\right). \quad (ii) \ x[n] = \cos\left(\frac{14\pi n}{15}\right).$$

Solution. (i) Periodic. The radian frequency is $\omega_0 = 4/9$, hence $T_0 = 2\pi/\omega_0 = 9\pi/2$. The phase does not affect the period.

(ii) Periodic. Write $\cos(14\pi n/15) = \cos(2\pi \frac{7}{15}n)$, so $f_0 = 7/15$. Since 7 and 15 are coprime, the fundamental period is $N = 15$.

(b) Simplify/evaluate the following expressions.

$$(i) \ \sum_{n=-\infty}^{\infty} (n^2 + 3n + 5) \delta[n + 1]. \quad (ii) \ \int_{-\infty}^{\infty} \delta(t - 2) e^{t-2} \cos\left(\frac{\pi}{6}(t^2 - t)\right) dt.$$

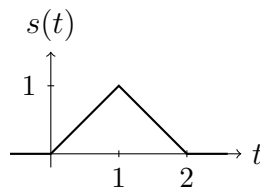
Solution. (i) The delta is nonzero only at $n = -1$, so the sum is $(-1)^2 + 3(-1) + 5 = 3$.

(ii) By the sifting (integration) property, evaluate the rest of the integrand at $t = 2$: $e^0 \cos\left(\frac{\pi}{6}(4 - 2)\right) = \cos(\pi/3) = \frac{1}{2}$.

Question 4 (Step response and convolution.)

An LTI system has the triangular step response

$$s(t) = \begin{cases} t, & 0 \leq t < 1, \\ 2 - t, & 1 \leq t < 2, \\ 0, & \text{otherwise,} \end{cases}$$

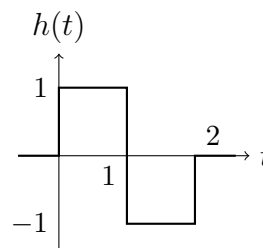


i.e., the output when the input is $u(t)$.

(a) Determine the impulse response $h(t)$, and sketch it.

Solution. Since $s(t) = \int_{-\infty}^t h(\tau) d\tau$, we have $h(t) = \frac{d}{dt}s(t)$. The slope of $s(t)$ is $+1$ on $(0, 1)$, -1 on $(1, 2)$, and 0 elsewhere; $s(t)$ has no jumps, so there are no deltas. Hence

$$h(t) = u(t) - 2u(t-1) + u(t-2) = \begin{cases} 1, & 0 \leq t < 1, \\ -1, & 1 \leq t < 2, \\ 0, & \text{otherwise.} \end{cases}$$



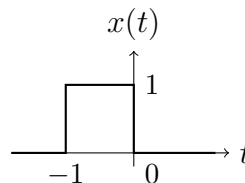
Note: If you are unsure of your answer to part (a), you may instead use $h(t) = u(t) - u(t-2)$ in parts (b) and (d). In that case, your answer to part (d) will not agree with your answer to part (c).

(b) Using $h(t)$, determine whether the system is memoryless and causal.

Solution. Not memoryless, since $h(t)$ is not of the form $A\delta(t)$. Causal, since $h(t) = 0$ for $t < 0$. (The same answers hold for the alternative $h(t) = u(t) - u(t-2)$.)

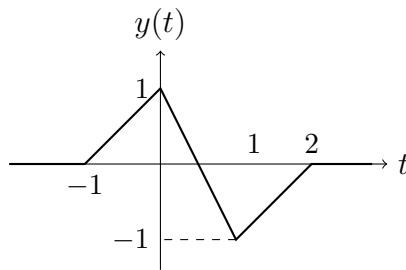
(c) Let the input be

$$x(t) = u(t+1) - u(t) = \begin{cases} 1, & -1 \leq t < 0, \\ 0, & \text{otherwise,} \end{cases}$$



Find $y(t)$ using only $s(t)$ together with linearity and time invariance, and sketch $y(t)$.

Solution. Since $x(t) = u(t+1) - u(t)$, linearity and time invariance give $y(t) = s(t+1) - s(t)$. For $-1 \leq t < 0$: $s(t+1) - s(t) = (t+1) - 0 = t+1$. For $0 \leq t < 1$: $(2 - (t+1)) - t = 1 - 2t$. For $1 \leq t < 2$: $0 - (2 - t) = t - 2$. Elsewhere both terms are zero.



(d) Now compute $y(t)$ by graphical convolution, flipping and sliding $x(t)$, i.e.,

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau.$$

Determine the ranges of t for which the formula for $y(t)$ is the same. For each range, set up the integral with explicit limits and evaluate it. Label at least one value on each axis of your plots.

Solution. $x(t - \tau) = 1$ when $-1 \leq t - \tau < 0$, i.e., for $t < \tau \leq t + 1$; this is a unit-height window of width 1 whose left edge is at $\tau = t$. Hence $y(t) = \int_t^{t+1} h(\tau) d\tau$, and the cases are determined by where the window $[t, t + 1]$ sits relative to the breakpoints 0, 1, 2 of h :

- $t < -1$: no overlap, $y(t) = 0$.
- $-1 \leq t < 0$: window overlaps $[0, t + 1]$ where $h = 1$: $y(t) = \int_0^{t+1} 1 d\tau = t + 1$.
- $0 \leq t < 1$: $y(t) = \int_t^1 1 d\tau + \int_1^{t+1} (-1) d\tau = (1 - t) - t = 1 - 2t$.
- $1 \leq t < 2$: window overlaps $[t, 2]$ where $h = -1$: $y(t) = \int_t^2 (-1) d\tau = t - 2$.
- $t \geq 2$: no overlap, $y(t) = 0$.

Check continuity: $y(-1) = 0$, $y(0) = 1$, $y(1) = -1$, $y(2) = 0$ from both sides. This matches part (c).

With the alternative $h(t) = u(t) - u(t - 2)$, which equals 1 on $[0, 2)$, the same window gives: $y(t) = 0$ for $t < -1$; $y(t) = t + 1$ for $-1 \leq t < 0$; $y(t) = \int_t^{t+1} 1 d\tau = 1$ for $0 \leq t < 1$; $y(t) = \int_t^2 1 d\tau = 2 - t$ for $1 \leq t < 2$; $y(t) = 0$ for $t \geq 2$.