

Grand Scheme

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Here is a description of how the topics of ECE 421 relate to each other, this is the “grand scheme” of the course.

ECE 421 begins with a review of *signals* and *systems*, with an emphasis on *sinusoidal signals* and *linear time invariant* (LTI) systems. LTI systems have two key advantages: (i) many real-world systems are LTI or approximately LTI; and (ii) LTI systems have many convenient mathematical properties. Therefore, the course studies LTI systems in great detail. *Indeed, the course revolves around two key properties of LTI systems and the sampling theorem.*

Sampling theorem: The sampling theorem states that sampling an analog signal at the Nyquist rate allows to perform all computations digitally before the signal is converted back to analog. Seeing that digital hardware has become exponentially cheaper and more powerful over the last several decades, ECE 421 emphasizes digital analysis and processing of signals and systems. The key components of most modern signal processing system are therefore:

- *A/D conversion* - we discuss the sampling theorem, especially that the signal must be sampled at the Nyquist rate.
- *Digital signal processing* – the main goal of the course is to provide an introduction to the analysis and design of these systems. Several such systems are described later.
- *D/A conversion* - the sampling theorem also guarantees perfect reconstruction, and we study ideal and practical interpolation.

First property: The first key property of LTI systems is that when sinusoidal signals are processed by LTI systems, they are amplified by some magnification factor. Consequently, it is especially insightful to *decompose signals into their harmonics*. And later, due to linearity, the amplification of each of the harmonics is summed at the output.

We study in detail the analysis of signals and systems in both the time and frequency domains, including the introduction of the *z-transform* and several varieties of *Fourier transforms*. These transforms allow to move between time and frequency – and back. Properties of Fourier transforms are discussed in detail, in order to deepen the understanding of when it is more convenient to work in the time domain or the frequency domain.

Second property: The second property of LTI systems is that they can be represented by a *convolution* between the LTI system’s impulse response and its input. Therefore, LTI

systems can be implemented digitally using convolution, and we study convolution and its properties, including how to implement it rapidly.

What do we do with these properties? Utilizing these properties of LTI systems, and the analysis and design tools that were developed, we can study several common types of digital signal processing systems.

- In many situations, some harmonic component of a signal is of interest. In order to extract harmonic components of interest from a signal, we develop *digital filters*, and discuss how to implement them efficiently.
- In other cases, we may want to *analyze the frequency content* of a signal. We learn how the Fourier transform can facilitate this type of processing.
- One way to implement a digital LTI system relies on *difference equations* – we analyze them and discuss how to implement them.
- Another approach to implement LTI systems relies on convolution.

We conclude the course by noting that Fourier transformation plays a crucial role in many digital signal processing systems, but for a signal of length N a naive implementation of the transform requires computation that is quadratic in N . Quadratic runtime is unacceptable in many problems of practical interest. The *fast fourier transform* (FFT) drastically reduces the runtime required for Fourier transformation to $N \log(N)$. We describe how to use the FFT, and outline how it works.

Throughout the course, we sprinkle in projects that tie the ideas to applications. And we also briefly present some modern signal processing techniques.