ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 1

Dror Baron; Spring 2023
Due date – January 24, 2024

Administrative instructions:

1. The homework must be submitted individually or in pairs.
2. You should submit electronically on Moodle by midnight the day that the homework is due.
3. Scans of handwritten answers are fine.
4. Please justify your answers carefully.

Question 1

In class on January 10, we analyzed the 4 possible functions of 1 bit. For $f_1(X) = 0$, $f_2(X) = 1$, and $f_3(X) = X$, we showed that $U_f : (X, Y) \to (X, Y \oplus f(X))$ is a one to one function, because it maps from all 4 pairs, $(X, Y) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ to the same 4 pairs (perhaps in a modified order), implying that the modified function U_f is invertible.
The last function, which we did not analyze, is $f_4(X) = NOT(X)$.
(a) Is f_4 reversible? Make sure to justify your answer.
(b) Now consider the modified function, U_{f_4}. Show that it is reversible.

Question 2

Consider the discrete time system,

$$y(n) = 0.5y(n-1) + x(n).$$

(a) What is the impulse response of this system? That is, if the input is $x(n) = \delta(n)$, what is the output, $y(n)$?
(b) Repeat part (a) for a step input, $x(n) = u(n)$.

Question 3

In class, we claimed that it suffices to prove the following two properties to prove linearity.

1. $\forall x_1, x_2$ sequences, $S[x_1 + x_2] = S[x_1] + S[x_2]$.
2. $\forall x_1$ sequence, $\forall \alpha \in \mathbb{R}, S[\alpha x_1] = \alpha S[x_1]$.

Prove that if the following two properties hold, then the system S is linear, meaning that
$\forall x_1, x_2$ sequences, $\forall \alpha, \beta \in \mathbb{R}, S[\alpha x_1 + \beta x_2] = \alpha S[x_1] + \beta S[x_2]$.