ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 2

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Due date – February 7, 2024

Administrative instructions:

1. The homework must be submitted individually or in pairs.
2. You should submit electronically on Moodle by midnight the day that the homework is due.
3. Scans of handwritten answers are fine.
4. Please justify your answers carefully.

Question 1
To help you appreciate the constants involved in quantum computers, compare the following two systems.

- A classical computer processes a size-N problem ($N = 2^n$) using $100N$ operations. The classical computer uses a state-of art GPU system that performs 10^{12} operations per second.
- A quantum computer processes the problem using n^3 operations. While the quantum machine performs 10^6 physical operations per second, various overheads such as using quantum error correction to reduce the impact of noise, reduce the effective speed to 10^3 operations per second.

The classical computer is faster for small n, and the quantum computer for large n. What is the crossover point where their speeds are comparable? (Hint: You do not need a precise answer; try out round numbers such as $(n = 10, N = 10^3), (n = 20, N = 10^6)$, and so on.) The idea in this question is to show that even an impressive exponential speedup will only be beneficial on large problems.)

Question 2
Prove Euler's identity,

$$\exp(i\theta) = \cos(\theta) + \sin(\theta)i.$$

To do so, use Taylor series expansions for $f_1(x) = \exp(x)$, $f_2(x) = \sin(x)$, and $f_3(x) = \cos(x)$. You may use the results we derived in class, although if you are rusty with this material, working through the entire question will be advantageous.

Question 3
Recall our formula for an infinite geometric series,

$$S(x) = \sum_{n=0}^{+\infty} x^n = \frac{1}{1-x},$$

provided that $|x| < 1$. In class, we derived this result by comparing $S(x)$ and $xS(x)$. Please derive the Taylor series for $\frac{1}{1-x}$ and confirm that it coincides with $\sum_{n=0}^{+\infty} x^n$.

Question 4
Now that you are proficient with Euler's identity, let's use it to show that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$. (Hint: You may spot an expression of the form $(a+b)(a-b)$, which equals $a^2 - b^2$, in your derivation.)

Question 5
In class we considered the following matrix, $A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$. Express this matrix in the $|+\rangle$ and $|-\rangle$ basis. That is, if the coordinates of some vector $|\psi\rangle$ are given with respect to (w.r.t.) the $|+\rangle$ and $|-\rangle$ basis, i.e., $|\psi\rangle = \alpha\,|+\rangle + \beta\,|-\rangle$, then find γ and δ such that $A\,|\psi\rangle = \gamma\,|+\rangle + \delta\,|-\rangle$, where γ and δ should be linear functions (meaning matrix multiplication) of α and β.