ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 3

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Due date – February 14, 2024

Administrative instructions:

1. The homework must be submitted individually or in pairs.
2. You should submit electronically on Moodle by midnight the day that the homework is due.
3. Scans of handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (matrix composition; page 64 in Nielsen and Chuang.)
Suppose that V, W, and X are linear vector spaces, and $A : V \rightarrow W$ and $B : W \rightarrow X$ are linear operators. We use the notation BA to denote the composition of A and B, meaning that $(BA)\left|\psi\right\rangle = B(A(\left|\psi\right\rangle))$. Show that the matrix representation of the BA linear operator is the product of the 2 matrices, A and B, that correspond to these linear operators. (To do so, you may want to define bases for V, X, and W.)

Question 2
Recall the Cauchy-Shwartz inequality, $|\left\langle v|w\right\rangle|^2 \leq \left\langle v|v\right\rangle \left\langle w|w\right\rangle$. When does equality hold?

Question 3
In class, we saw that the eigen-values of the Pauli-X and Pauli-Z matrices are ± 1. Compute the eigen-values and eigen-vectors for one of the other "standard" matrices, Y or H (Hadamard).

Question 4 (Exercise 2.24 in Nielsen and Chuang.)
Show that a positive operator is Hermitian. (Hint: show that any linear operator A can be written as $A = B + iC$ where B and C are both Hermitian.)

Question 4
Compute the square root of the following matrix,

$$A = \begin{bmatrix} 6.5 & 2.5 \\ 2.5 & 6.5 \end{bmatrix}.$$