ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 5

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Due date – February 28, 2024

Administrative instructions:

1. The homework must be submitted individually or in pairs.
2. You should submit electronically on Moodle by midnight the day that the homework is due.
3. Scans of handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (DFT.)
Compute the 4-point discrete Fourier transform (DFT) for the periodic sequence, $x_(n) = [\underline{0}, 1, 2, 3]$. Your answer should have some conjugate symmetry property, please point it out.

Question 2 (Circular convolution.)
The Fourier transform for aperiodic sequences obeys $x_1 * x_2 \leftrightarrow X_1(\omega)X_2(\omega)$. For periodic sequences, the circular convolution between x_1 and x_2, denoted by $x_1 \otimes x_2$, has a DFT $X_1(k)X_2(k)$. Consider two periodic-$N = 2$ signals, $x_1 = [\underline{1}, 1]$ and $x_2 = [\underline{1}, 2]$. Compute their circular convolution using the following steps: (i) compute the DFTs of x_1 and x_2; (ii) multiply the DFTs, element by element; (iii) find a periodic-$N = 2$ signal whose DFT equals $X_1(k)X_2(k)$.

Question 3 (Exponent of matrix.)
Let θ be a real number. Show that

$$\exp(i\theta X) = \cos(\theta)I + i\sin(\theta)X, \tag{1}$$

where X is the Pauli-X matrrix, and I is the 2×2 identity matrix. (Ths question is based on Exercise 4.2 in Nielsen and Chuang. It combines matrix operators from our linear algebra segment of material with ideas that will be used when we study quantum gates. Instead of X, you can prove the identity for any matrix A such that $A^2 = A$. Finally, as a hint, there are at least two ways to show (1). The first is to look at the diagonalized form of $X = (1)\left|+\right\rangle\left\langle+\right| + (-1)\left|-\right\rangle\left\langle-\right|$, and apply $\exp(i\theta\lambda$ to each eigen value, λ. The second way is to consider the Taylor series for exp, sin, and cos that you used to prove Euler's identity in Homework 2.)