ECE 592–100 – Signal Processing Tour of Quantum Computing
Homework 6

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Due date – March 6, 2024

Administrative instructions:

1. The homework must be submitted individually or in pairs.
2. You should submit electronically on Moodle by midnight the day that the homework is due.
3. Scans of handwritten answers are fine.
4. Please justify your answers carefully.

Question 1 (Measurement operators.)
Recall that $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$ are projection matrices. Consider using the following set of measurement operators, M_0, $0.8M_1$, and $0.6M_1$.
(a) Show that the completeness relation holds, i.e., $\sum_m M_m^\dagger M_m = I$.
(b) For $|\psi\rangle = [\alpha\ \beta]^T$, compute the probabilities of measuring M_0, $0.8M_1$, and $0.6M_1$.

Question 2 (Observables.)
Consider the observable matrix, $M = 2|+\rangle\langle +| - |-\rangle\langle -|$.
(a) Is M is Hermitian? Unitary? Normal? Positive?
(b) What is the average measurement if $|\psi\rangle$ is $|0\rangle$?

Question 3 (Parallel quantum operators.)
Consider 2 qubits, $|\psi_1\rangle = [\alpha\ \beta]^T$ and $|\psi_2\rangle = [\gamma\ \delta]^T$. We now apply X to the first qubit and Z to the second qubit. These operations can be performed in parallel, meaning that the following systems have the same outputs: (*i*) form $|\psi_3\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, then apply X to get $|\psi_4\rangle$, then apply Z to get $|\psi_5\rangle$; (*ii*) similar to above, but swap the order of X and Z; and (*iii*) apply X and Z together, meaning that we go from $|\psi_3\rangle$ directly to $|\psi_5\rangle$. Your role is to show that $|\psi_5\rangle$ is identical using all 3 approaches.

Question 4 (Controlled Z gate.)
(This is exercise 4.18 in Nielsen & Chuang.) Show that applying a controlled Z operation to the first qubit is identical to applying a controlled Z to the second one. To make things more concrete, applying a controlled Z to the second qubit means that if the control qubit (the first one) is 1, then we apply Z to the second target qubit, else the second qubit is unchanged (we apply I). Applying a controlled Z to the first qubit means that the roles of the qubits are changed.