ECE 592-100 – Signal Processing Tour of Quantum Computing

Quiz 1– Spring 2024

February 7, 2024

Please remember to justify your answers carefully.

Last name: ____________________ First name: ____________________

Question 1 (Orthogonality.)
Consider the row vector, $v_1 = [1\ 2]$. Describe a second row vector, v_2, which is nonzero and orthogonal to v_1.

Solution. One solution is $v_2 = [2\ \ -1]$. The inner product between the vectors is

$$(v_1, v_2) = 1 \times 2 + 2 \times (-1) = 0,$$

hence they are orthogonal.

Question 2 (Diagonalizable matices.)
Consider the matrix

$$A = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}.$$

Show that A is not diagonalizable.

Solution. It is easily seen that AA^+ and A^+A are different, where $A^+ = A^T$, because A is real-valued. Because $AA^T \neq A^TA$, A is not normal, hence it is not diagonalizable.

Question 3 (Eigen values of matrix.)
Consider the following matrix,

$$A = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}.$$

(a) Compute the eigen values of A.

Solution. Taking the determinant of $A - \lambda I$, the characteristic polynomial is

$$(0 - \lambda)(0 - \lambda) - (4)(1) = \lambda^2 - 4 = (\lambda - 2)(\lambda + 2).$$

The roots of this polynomial are the eigen values, $\lambda_1 = -2$ and $\lambda_2 = +2$.

(b) Can A be a projection matrix? Make sure to justify your answer.

Solution. Projection matrices have eigen values that are either 0 or 1. Both eigen values from part a are neither 0 nor 1, hence A is not a projection matrix.

(c) Can A be a positive matrix? Make sure to justify your answer.

Solution. Positive and positive definite matrices have non-negative (respectively, positive) eigen-values. In any event, the negative eigen value, $\lambda_1 = -2$ from part (a), rules out the possibility that A is positive.