ECE 592-100 – Signal Processing Tour of Quantum Computing

Quiz 2– Spring 2023

March 6, 2023

Please remember to justify your answers carefully.

Last name: ____________________ First name: ____________________

Question 1 (Period of signal.)
Consider the discrete time signal, $x(n) = cos(0.1\pi n) + \cos(0.2\pi n)$. What is the period of this signal?

Solution. The period of $\cos(0.1\pi n)$ is $N_1 = 20$, because $20 \times 0.1\pi = 2\pi$ radians per sample, or one cycle per sample. Similarly, the period of $\cos(0.2\pi n)$ is $N_1 = 10$. The overall period is 20, because N_1 and N_2 both divide 20.

Question 2 (Fourier using different periods.)
Consider a period discrete time signal where the period is $N = 4$, the samples are $x = \{\underline{1}\ 0\ 1\ 0\}$, and the underline corresponds to time index $n = 0$.
(a) Compute the Fourier coefficients, C_k, for $k \in \{0, 1, 2, 3\}$.
(b) Note that x is also periodic-2, because $x(0) = x(2)$ and $x(1) = x(3)$. Using $N = 2$, re-compute the Fourier coefficients, C_k, for $k \in \{0, 1\}$.

Solution. (a) $C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-i2\pi kn/N} = \frac{1}{4}\left[e^{-i2\pi k0/4} + e^{-i2\pi k2/4}\right] = \frac{1}{4}(1+(-1)^k)$, which is $\frac{1}{2}$ for $k = 0, 2$, and 0 for $k = 1, 3$. (In this part, $N = 4$.)
(b) In this part, $N = 2$, and $C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-i2\pi kn/N} = \frac{1}{2}e^{-i2\pi k0/2} = \frac{1}{2}$ for $k = 0, 1$.

Question 3 (LTI systems with periodic inputs.)
Consider an input signal, $x(n)$, that has period N. The signal is processed by a linear time invariant (LTI) system whose impulse response is h. You will see in this question that y is also periodic.
(a) Recall that we can express $x(n)$ as a linear combination over exponential signals,

$$x(n) = \sum_{k=0}^{N-1} C_k e^{+i2\pi kn/N} = \sum_{k=0}^{N-1} C_k x_k(n),$$

where $x_k(n) = e^{+i2\pi kn/N}$ and $k \in \{0, \ldots, N-1\}$. Each $x_k(n)$ can be an input to H, and corresponds to an output, $y_k(n)$. Express $y_k(n)$ using $x_k(n)$ and $H(\omega)$.

(b) Using linearity, what is the output, $y(n)$, of the LTI system for $x(n)$? (Do not express your answer as convolution between x and h. Rely on part (a).)
(c) Why is your answer to part (b) periodic-N? (Its period might be smaller than N as in Question 2; but it will satisfy $y(n) = y(n+N)$.)

Solution. (a) $x_k(n) = e^{+i2\pi kn/N}$, and the corresponding output, $y_k(n)$, multiplies $x(k)$ by $H(\omega = 2\pi k/N)$, hence $y_k(n) = x_k(n)H(\omega = 2\pi k/N)$.
(b) $y(n) = \sum_{k=0}^{N-1} C_k y_k(n) = \sum_{k=0}^{N-1} C_k H(\omega = 2\pi k/N)e^{i2\pi kn/N}$.
(c) Each of the sinusoidal components, $y_k(n)$, in part (b) is N-periodic, hence the linear combination of $y_k(n)$ is also N-periodic.